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Editorial: Quasi-normal modes, non-selfadjoint operators and pseudospectrum: an interdisciplinary approach

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Editorial on the Research Topic
Quasi-normal modes, non-selfadjoint operators and pseudospectrum: an interdisciplinary approach

In recent years, quasi-normal mode frequencies, namely, the complex numbers encoding the linear response of "damped" resonators to external perturbations have acquired major importance in different settings of physics, ranging from astrophysical and theoretical problems in gravitational physics to the study of the scattering properties of optical nanoresonators. Beyond physics, this subject is directly related to the study of the spectral and dynamical properties of non-selfadjoint operators, a very active area of research in applied and fundamental mathematics with direct applications in physics, ranging from hydrodynamics and turbulence to non-Hermitian quantum mechanics. Despite these complementary interests and converging "working knowledge", research interactions among the involved subcommunities appear to be quite scarce. This Research Topic represents an effort to bring attention to the interdisciplinary nature of the research around the notion of quasi-normal modes and non-Hermitian -or non-selfadjoint- dynamics. As mentioned, this interdisciplinarity operates at multiple levels, extending from the dialogue between physics and mathematics to the interchanges among the different subcommunities within these two disciplines.

The notion of normal mode pervades physics, providing a common conceptual and technical thread among different subfields of research and offering a basis for the study of (conservative) linear dynamics. The key mathematical property underlying normal modes is the diagonalizability of self-adjoint operators in terms of an orthonormal basis, which is guaranteed by the "spectral theorem". Its validity extends to operators that commute with their adjoints, namely, "normal operators". Normal modes and their associated spectrum ("normal frequencies") stand as a cornerstone of the dynamics driven by such normal operators.

A fundamental change occurs in non-conservative systems driven by non-selfadjoint operators or, more generally, non-normal operators. Familiar normal modes are then substituted by "quasi-normal modes" (QNMs), which encode in an invariant manner the characteristic linear response of a system to external perturbations and indeed share some of the features of normal modes. However, the loss of the spectral theorem critically impacts QNMs: their completeness is not guaranteed, their orthogonality is lost, and the corresponding eigenvalues in the spectrum are potentially unstable under small perturbations. "Non-normal dynamics" (Trefethen and Embree [1]) driven by these operators are then subject to characteristic non-normal effects that are absent in the normal case, such as spectral instabilities in QNM frequencies, growth transients, or pseudo-resonances [2]. These differences ultimately respond to a key contrast between the normal and non-normal operator theories: the respective structural status of the spectrum. Whereas the spectrum of normal (time-generator) operators provides a tight control of the full dynamics, in the non-normal case such control is not guaranteed by the spectrum alone—except for late dynamics—and specific tools from non-selfadjoint spectral theory are required.

Non-modal analysis (e.g., Schmid [3]) provides a framework for the study of non-normal dynamics by crucially incorporating concepts and tools from the (spectral) theory of non-self-adjoint operators. This Research Topic highlights the notion of pseudospectra and their relation to QNMs and their properties. Pseudospectra sets in the complex frequency plane contain, notably, the (QNM) spectrum set, but also encode more information crucial to seize all of the dynamics, in particular, the above-mentioned non-normal effects. However, whereas the spectrum concept is built only on the operator itself, the pseudospectra depend on the choice of scalar product and associated norm. The same applies to other key non-modal analysis tools, in particular the "growth function", which is crucial in the study of non-modal growth transients and the assessment of optimal disturbances (see below). The question of the choice of the scalar product becomes a central theme in the non-modal analysis of non-normal dynamics, and in particular in this Research Topic.

We will now present the articles in this Research Topic. Given the interrelation among the contributions, the arrangement by category is somewhat "*ad hoc*", but we hope it illustrates the interdisciplinarity of the subject, both within the physics community and in its connection with ongoing mathematical developments.

1. Quasi-normal modes as an eigenvalue problem.

A key prerequisite for the application of non-normal dynamical concepts to scattering and QNMs is casting the dynamics in terms of a non-selfadjoint (non-normal) time generator. Building on the spacetime geometric insights developed in general relativity, Panosso Macedo and Zenginoğlu review the so-called hyperboloidal approach to scattering on black hole (BH) spacetimes. This geometric scheme provides the required non-selfadjoint operator, with non-normality being associated with losses through spacetime boundaries. BH QNMs are then cast as the eigenvalues of a non-selfadjoint spectral problem. Applications to QNM excitation coefficients, spectral QNM instability, and quadratic QNMs are presented. An interesting extension of such a hyperboloidal scheme to problems with dispersion (e.g., some quantum gravity-motivated problems) is presented in Burgess and König.

Another approach to casting resonances (QNMs) as an eigenvalue problem is the so-called complex scaling method (see, for example, Simon [4] and references therein). Although such a method is referred to in Warnick and Vogel, it is not really discussed in this Research Topic. However, Richarte et al. discuss a spectral problem reminiscent of such complex scaling.They review the approach to QNMs as analytical continuations of bound states of an appropriate self-adjoint operator and, in particular, highlight a non-selfadjoint spectral issue: the failure of the method to recover QNMs whose analytic continuation is not in the domain where the operator defining the bound states is selfadjoint.

2. Non-normal spectral and dynamical aspects in gravitational (black hole) physics.

The first indications of non-normal behavior of QNMs in gravity were implicit in the seminal results on BH QNM instabilities presented in Aguirregabiria and Vishveshwara [5]; Vishveshwara [6] and in Nollert [7]; Nollert and Price [8]. However, it was precisely the hyperboloidal approach, reviewed in Panosso Macedo and Zenginoğlu that allowed to establish transparently in Jaramillo et al. [9] the essential role of non-normal mechanisms. That was the starting point of the rapid development of non-normal dynamics in the gravitational context, one of the main subjects of this Research Topic.

2.i. Black hole QNM spectral (in)stabilities.

Warnick presents the first mathematically rigorous account in the literature of the BH QNM instability phenomenon. Characterized as a "perturbative" spectral instability, an analytical discussion in terms of QNM "modes" and "co-modes" is presented. Sensitivity to perturbations is enhanced as damping increases, a feature that is explained in terms of the need to control higher derivatives to properly define increasing QNM overtones. This feature can be (dually) interpreted as a *spectral stability in high-order Sobolev norms*or, equivalently, as a *spectral instability in the "energy norm"*, the consequence of distributing energy over small scales. A generalization of pseudospectra is introduced, tailored to the non-normality of the operators appearing in the BH scattering problem.

In a complementary work, Boyanov disentangles the confusion between the instability discussed in Warnick from another important BH QNM instability, referred to as the "flea on the elephant" (reminiscent of Simon [10]). These two spectral instabilities respond to distinct instability mechanisms; the first one corresponds to the "perturbation instability" of (already) existing QNMs, whereas the second one involves a "branch instability" with the appearance of a new family of QNMs.

Other aspects of pseudospectra and spectral instability are presented in the articles by Areán et al., Besson et al., Burgess and König, Drysdale and Johnson, Krejčiřík and Siegl, and Vogel.

2.ii. Non-modal growth transients in black holes.

Besson et al. review the first studies on non-modal growth transients in the gravitational setting, namely, for fields scattered on BHs. The discussion covers both frequency (pseudospectrum) and time-domain approaches, crucially presenting the first studies of the time-domain growth function $G(t)$ for BHs. The role of the norm choice for non-modal growth transients is highlighted, something that is further emphasized in Díaz Palencia, where the control of the dynamics of perturbations on BHs is discussed. These articles on growth transients pave the way for the systematic application of non-modal analysis to study non-normal dynamical effects in gravity. Forced systems—that in general relativity arise from second-order perturbation theory–leading to pseudo-resonances, would be a natural next step. Non-modal growth transients are also discussed in the article by Drysdale and Johnson.

3. QNMs and non-normality in high-energy physics.

Areán et al. review holographic duality, also referred to as the AdS/CFT correspondence or gauge/gravity duality, namely, a strong/weak coupling duality between a boundary gauge field theory and a bulk gravitational one. Focus is placed on holographic QNMs controlling the return to thermal equilibrium in the boundary gauge theory, providing a link to "linear response theory" and the Schwinger-Keldysh formalism. The spectral instability and pseudospectra of holographic QNMs are discussed. Non-modal transients in the holographic setting are also reviewed in Besson et al.

4. Quasi-normal modes in optics and plasmonics.

Wu and Lalanne review the remarkable development that QNM theory has recently experienced in optics and plasmonics. Focus is placed on the role of QNM theory in designing and understanding micro- and nano-resonators, which play a key role in current photonics, with an emphasis on the notions of "mode hybridization" and "mode perturbation". As an instance of optics-gravity interdisciplinarity, and motivated by the study of the fiber optical soliton, Burgess and König present an adaptation of the spacetime hyperboloidal approach to scattering and QNMs (see also Al Sheikh [11]) to settings with dispersion. This development is crucial in optics and in some dispersive modified gravity theories.

5. Mathematical aspects of non-normality.

We collect here articles from the Research Topic that cover mathematical subjects and range from pseudospectra, spectral instability and random perturbations, oriented graphs and networks, and partial differential equation (PDE) hyperbolicity. They harmoniously complement the physical contributions:

5.i. Krejčiřík and Siegl discuss the pseudospectrum from the perspective of "pseudomodes", a notion of approximate eigenvector that is not to be confused with QNMs, although it is relevant to the study of the spectral instability of the latter. In particular, a construction of pseudomodes for large "pseudoeigenvalues" is discussed using tools that do not rely on semi-classical analysis. This construction may be relevant in the context of the spectral instability of highly damped BH QNMs, as discussed above.

5.ii. Vogel presents a mathematical account of the pseudospectrum, focusing on three important topics: pseudospectra of semi-classical pseudodifferential operators; pseudospectra of random matrices; and eigenvalue asymptotics of non-selfadjoint random operators; (cf. also Sjöstrand [12]). Particularly important for the BH QNM spectral (perturbative) instability presented above is the discussion of eigenvalue asymptotics and "regularity improvement" for non-selfadjoint random operators.

5.iii. Drysdale and Johnson discuss the directionality of a directed graph, from the non-normality of the associated adjacency matrix. Then, the relation to the "trophic coherence" of a network is studied, and following the analysis, it is suggested to extend the notion of trophic coherence to matrices beyond the network setting, in particular in the context of (non-normal) dynamics and non-modal transients.

5.iv. Abalos et al. discuss the characterization of "strong hyperbolicity" in evolution PDE systems with constraints, in particular through "extensions" of the PDE system with additional variables. Strong hyperbolicity is then controlled by a condition involving the singular value decomposition of the (square matrix) principal symbol of the resulting PDE system. Studying the (non-)normality of the principal symbol may then be a good starting point for assessing the interplay of non-modal effects and hyperbolicity.

In summary, the study of scattering and QNMs from a non-normal dynamics perspective, by adopting a non-modal analysis approach, offers a rich arena for interdisciplinary research at the interface between physics and mathematics, and, within the physics realm, among gravity, optics, and hydrodynamics.

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References

1. Trefethen L, Embree M. *Spectra and pseudospectra: the behavior of nonnormal matrices and operators*. Princeton, NJ: Princeton University Press (2005).

2. Trefethen LN, Trefethen AE, Reddy SC, Driscoll TA. Hydrodynamic stability without eigenvalues. *Science* (1993) 261:578–84. doi:10.1126/science.261.5121.578

3. Schmid PJ. Nonmodal stability theory. *Annu Rev Fluid Mech* (2007) 39:129–62. doi:10.1146/annurev.fluid.38.050304.092139

4. Simon B. Resonances and complex scaling: a rigorous overview. *Int J Quan Chem* (1978) 14:529–42. doi:10.1002/qua.560140415

5. Aguirregabiria JM, Vishveshwara CV. Scattering by black holes: a simulated potential approach. *Phys Lett A* (1996) 210:251–4. doi:10.1016/0375-9601(95)00937-X

6. Vishveshwara CV. On the black hole trail. In: *18th Conference of the Indian Association for General Relativity and Gravitation Madras*. Madras, India: Institute of Mathematical Science Report, by Madras Univ. Inst. Math. Sci. (1996). p. 11–22.

7. Nollert H-P. About the significance of quasinormal modes of black holes. *Phys Rev D* (1996) 53:4397–402. doi:10.1103/PhysRevD.53.4397

8. Nollert H-P, Price RH. Quantifying excitations of quasinormal mode systems. *J Math Phys* (1999) 40:980–1010. doi:10.1063/1.532698

9. Jaramillo JL, Panosso Macedo R, Al Sheikh L. Pseudospectrum and black hole quasinormal mode instability. *Phys Rev X* (2021) 11:031003. doi:10.1103/PhysRevX.11.031003

10. Simon B. Semiclassical analysis of low lying eigenvalues. iv. the flea on the elephant. *J Functional Analysis* (1985) 63:123–36. doi:10.1016/0022-1236(85)90101-6

11. Al Sheikh L. Scattering resonances and pseudospectrum: stability and completeness aspects in optical and gravitational systems. Theses. Université Bourgogne Franche-Comté (2022). . Available online from: https://theses.hal.science/tel-04116011

12. Sjöstrand J. *Non-self-adjoint differential operators, spectral asymptotics and random perturbations Pseudo-Differential Operators*. New York: Springer International Publishing (2019). Available online from: https://www.springer.com/gp/book/9783030108182