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A quantum partial adiabatic evolution and its application to quantum search problem

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This paper presents a framework for quantum partial adiabatic evolution and applies it to re-examine the well-known quantum search problem. We particularly focus on a detailed analysis of the algorithm's success probability, which serves as a clear criterion for differentiating valid implementations from invalid ones. Specifically, when the time complexity aligns with the optimal quantum computation, the algorithm achieves a substantially high success probability. Conversely, so-called "improved" versions that exceed the quadratic speedup characteristic of quantum computing exhibit a negligibly low success probability with the increase of target elements. These findings underscore the critical importance of selecting the appropriate evolution interval and the correct method for calculating the success probability in studies of quantum partial adiabatic evolution.

KEYWORDS

quantum computation, quantum partial adiabatic evolution, quantum search, success probability, time complexity

1 Introduction

The framework of quantum adiabatic evolution Farhi et al. [1, 2] provides a Hamiltonian-based model of quantum computation that is computationally equivalent to the standard gate-based model [3, 4]. Its utility is demonstrated by the range of novel algorithms it has inspired [5–8], offering a critical approach in a field where designing efficient algorithms is notably difficult. The core premise, rooted in the quantum adiabatic theorem [9], is to prepare the system in the ground state of an initial Hamiltonian and then adiabatically evolve it into a problem-encoding final Hamiltonian. A sufficiently slow evolution ensures the system remains in the ground state with high probability, allowing the solution to be obtained by measurement.

In early studies [2, 10], it was observed that a direct adiabatic implementation of Grover's search problem yielded no quantum advantage over classical computation, in contrast to the quadratic speedup of the original Grover algorithm [11]. This limitation was addressed by the introduction of quantum local adiabatic evolution in [10, 12], which successfully recovered the quadratic speedup. Furthermore, it was proven that this performance represents the fundamental limit for quantum local adiabatic computation Das et al. [10]. Moreover, quantum local adiabatic evolution has found other applications, such as in the well-known Deutsch-Jozsa problem [13].

In Tulsi [14], Tulsi studied a class of quantum adiabatic evolutions where either the initial or final Hamiltonian is a one-dimensional projector onto its ground state. It was shown that the minimum energy gap governing the evolution time is proportional to the overlap

between the ground states of the initial and final Hamiltonians. Moreover, such evolutions can exhibit a rapid crossover near the point of minimum gap, where the ground state changes abruptly. This insight led to the proposal of a faster partial adiabatic evolution, confined to a narrow interval around the minimum gap point.

The problem of searching an unstructured database for a marked item is a fundamental task in computer science. Classically, this requires $O(N)$ queries to the database. In a seminal work, Grover demonstrated that quantum mechanics provides a quadratic speedup, solving the problem with only $O(\sqrt{N})$ queries [11]. This quantum advantage arises from the coherent amplification of the amplitude associated with the target state. Subsequently, this algorithm was adapted into the framework of quantum adiabatic computation [2]. A key development was the local adiabatic search algorithm by Roland and Cerf [10], which achieved the optimal time complexity of $T = O(\sqrt{N/M})$ for finding M target items. The critical insight of this approach is the strategic relaxation of the standard global adiabatic condition. The traditional adiabatic theorem mandates a slow evolution rate across the entire duration $s \in [0,1]$ to prevent transitions to any excited state. However, for the quantum search problem, the dynamics are effectively confined to a two-dimensional subspace where the minimum energy gap $\Delta_{\min}$, which dictates the necessary evolution time, occurs at a single point $s = s^*$. The partial adiabatic approach recognizes that it is sufficient to enforce the adiabatic condition only near this avoided crossing $s \approx s^*$, where the gap is small and transitions are most likely. Away from this critical region, the system can be evolved much more rapidly. This focused application of the adiabatic condition leads to Tulsi's proposal of quantum partial adiabatic evolution [14]. The works of Zhang et al. [15, 16] further explored this framework to study quantum search problem. It was established a time complexity of $T = O(\sqrt{N}/M)$ for finding M target items in a database of size N [15], which achieves an $O(\sqrt{M})$ improvement over local adiabatic search. It retains a square-root speedup over classical search even for a single target Zhang et al. [16]. In Sun et al. [17], we introduced a quantum micro-local adiabatic search, a refinement in which the local adiabatic evolution is confined to a narrow interval, in contrast to a global evolution spanning the entire parameter range. However, it exhibited the same asymptotic scaling as earlier partial adiabatic schemes [15, 16], namely, with a time complexity of $O(\sqrt{N}/M)$, suggesting their optimality. Furthermore, in Sun et al. [18], we demonstrated that both quantum global and local adiabatic computation can be recovered from the partial adiabatic evolution by appropriately adjusting the evolution interval.

Nevertheless, the claimed $O(\sqrt{N}/M)$ complexity raises concerns, as it appears to contradict the established optimality of quadratic quantum speedup [10, 19]. Kay first identified this discrepancy and pointed out an oversight in Tulsi's original proof [20]. He showed that while the argument in Tulsi [14] could be corrected to validate the scheme, the same recovery is not generally possible for subsequent studies [15–17, 18, 21], leaving their conclusions in doubt.

Motivated by Tulsi's work and aiming to simplify the problem setting, this paper introduces a framework for quantum partial adiabatic evolution and investigates its application to quantum search. A central focus of our analysis is the rigorous evaluation of the algorithmic success probability. The main conclusions are as follows. Firstly, a valid partial search algorithm, whose time complexity is consistent with the fundamental limits of quantum computation, can achieve a high success probability, provided the constant defining the evolution interval is chosen sufficiently large. Conversely, in certain "improved" partial adiabatic search schemes [15, 16], as the number of the targets increases, the success probability is found to be remarkably small. This dichotomy establishes a clear demarcation between valid and invalid quantum partial adiabatic computations and underscores the critical importance of both the selection of the evolution interval and the accurate computation of success probability.

The organization of this paper is as follows. In Section 2, the proposed framework for quantum partial adiabatic evolution is detailed. Section 3 is devoted to the analysis of the quantum search problem within this framework, including comprehensive derivations of the success probability for both the valid algorithm and its invalid counterparts. The paper concludes with a summary and discussion in Section 4.

2 The framework of quantum partial adiabatic evolution

We define the system Hamiltonian as

$$H(s) = (1-s)H_i - sH_f, \tag{1}$$

parametrized by $s \in [0,1]$. The initial and final Hamiltonians are given by

$$H_i = I - |\alpha\rangle\langle\alpha|, H_f = |\beta\rangle\langle\beta|, \tag{2}$$

The parameter $s(t)$ evolves with time from $s(0) = 0$ to $s(T) = 1$.

The problem setting of Equation 1 with Equation 2 in this work is closely aligned with that of [14]. However, following the crucial insight from Kay [20], our method for calculating the success probability of the quantum partial adiabatic evolution is fundamentally distinct. Crucially, for any finite constant defining the evolution interval, the difference between the two resulting success probabilities is strictly greater than zero. This critical point will be elucidated soon in this section.

It is known that a standard quantum adiabatic algorithm for the above problem requires a time complexity of $T = O(a^{-2})$ [2, 10], while a quantum local adiabatic search achieves $T = O(a^{-1})$, where $a = |\langle\alpha|\beta\rangle|$ [10]. The goal of quantum partial adiabatic evolution is to achieve the same quadratic speedup over classical computation as the local adiabatic approach, but without requiring a finely-tuned, time-dependent evolution rate $\mathrm{d}s/\mathrm{d}t$. The main procedure of this method can be summarized as follows.

1. Initialize the system in the known ground state $|\alpha\rangle$.
2. Evolve the system adiabatically by sweeping the parameter s from $s^- = \frac{1}{2} - \delta$ to $s^+ = \frac{1}{2} + \delta$.
3. Measure the final state in the computational basis and verify if the outcome is a solution.

These steps are repeated until a marked state is found. The parameter δ is tunable; in our study of quantum partial adiabatic evolution, we set $\delta = ca$ for a positive constant c.

Before presenting the time complexity analysis, we begin by calculating the success probability of a single round of the

quantum partial adiabatic evolution. For this, as suggested in Kay [20], We should first verify that the overlap between the initial state and the eigenstate at s^- is sufficiently large. Following [20], the verification condition is given by the inequality

$$|\langle E_0(s^-)|\alpha\rangle| > 1/\sqrt{2}, \tag{3}$$

where $E_0(s)$ denotes the ground state of $H(s)$. Having established this, our next objectives are to determine the two lowest eigenvalues and the ground state of $H(s)$.

The initial state of the system is prepared within $|\alpha\rangle, |\beta\rangle$, and the action of the Hamiltonian $H(s)$ throughout the adiabatic evolution only induces transitions between $|\alpha\rangle$ and $|\beta\rangle$, without coupling to states outside this subspace. This is because states orthogonal to this subspace belong to different symmetry sectors or have vastly different energies. Thus, the Hamiltonian effectively acts as the identity on the orthogonal subspace, and the relevant dynamics are entirely captured by the two-dimensional model. So we restrict to the subspace spanned by $|\alpha\rangle$ and the part of $|\beta\rangle$ orthogonal to $|\alpha\rangle$. Define an orthonormal basis $|\alpha\rangle$ and $|\beta'\rangle$ with $|\beta\rangle = a|\alpha\rangle + b|\beta'\rangle, b = \sqrt{1-a^2}$. In this basis, the matrix representation of $H(s)$ is given by

$$H(s) = \begin{pmatrix} -sa^2 & -sab \\ -sab & 1-2s+sa^2 \end{pmatrix}. \tag{4}$$

The eigenvalues E of Equation 4 satisfy the characteristic equation $\det(H(s) - EI) = 0$:

$$\det\begin{pmatrix} -sa^2 - E & -sab \\ -sab & 1-2s+sa^2 - E \end{pmatrix}. \tag{5}$$

Computing the determinant in Equation 5

$$(-sa^2 - E)(1-2s+sa^2 - E) - (-sab)(-sab) = 0. \tag{6}$$

Thus, the characteristic Equation 6 becomes

$$E^2 + (2s-1)E - sa^2(1-s) = 0. \tag{7}$$

Solving the quadratic Equation 7, we can get the eigenvalues of $H(s)$, i.e.,

$$E_{0,1}(s) = \frac{1-2s\mp\Delta}{2}, \Delta = \sqrt{1-4s(1-s)(1-a^2)}. \tag{8}$$

We next seek the ground state

$$|E_0(s)\rangle = \cos\theta|\alpha\rangle + \sin\theta|\beta'\rangle. \tag{9}$$

Substituting Equation 9 into the eigenvalue equation $H(s)|E_0(s)\rangle = E_0|E_0(s)\rangle$ for $E_0 = E_0(s)$,

$$\begin{pmatrix} -sa^2 & -sab \\ -sab & 1-2s+sa^2 \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = E_0\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{10}$$

Equation 10 gives two equations

$$-sa^2\cos\theta - sab\sin\theta = E_0\cos\theta, -sab\cos\theta + (1-2s+sa^2)\sin\theta = E_0\sin\theta. \tag{11}$$

From these two equations in Equation 11, it can be verified that

$$\tan\theta = -\frac{E_0 + sa^2}{sab}, \tag{12}$$

Equation 12 together with the equality $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ leads to that

$$\cos(2\theta) = \frac{1-2sb^2}{\Delta}, \sin(2\theta) = \frac{2sab}{\Delta}. \tag{13}$$

By the equations in Equation 13, the following equality is easy to obtain

$$|\langle E_0(s)|\alpha\rangle| = \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{\Delta+1-2b^2s}{2\Delta}}, \tag{14}$$

and the equality (Equation 3) is verified directly,

$$|\langle E_0(s^-)\,|\,\alpha\rangle| = \sqrt{\frac{1}{2} + \frac{1}{\sqrt{1+4c^2(1-a^2)}}\left(\frac{a}{2} + c(1-a^2)\right)} > 1/\sqrt{2}. \tag{15}$$

Denote P as the success probability of one round of quantum partial adiabatic evolution. It has been corrected and can be calculated from Equation 15 as follows

$$P = (2|\langle E_0(s^-)|\alpha\rangle|^2 - 1)^2 = \frac{[a+2c(1-a^2)]^2}{1+4c^2(1-a^2)}. \tag{16}$$

Then it can be found out that

$$P \approx \frac{4c^2}{1+4c^2} \tag{17}$$

for $a \ll 1$ by some direct calculations.

Our next step is to show an analysis of the time complexity of the quantum partial adiabatic evolution. For this, we adopt the following formula which is also used in the prior works like Sun et al. [22] and Mei et al. [23] for the one round time cost estimation, defined as the duration needed to evolve the system from the initial state at s^- to the final state at s^+,

$$T' \geq \frac{s^+ - s^-}{\Delta_{\min}^2}, \tag{18}$$

in which

$$\Delta_{\min} = \min_{s\in[s^-,s^+]}(E_1(s) - E_0(s)). \tag{19}$$

By Equation 8, it can be inferred that $\Delta_{\min} = a$ from Equation 19. Meanwhile, by noting that $s^- = \frac{1}{2} - ca, s^+ = \frac{1}{2} + ca$, we are led to that $T' \geq 2ca^{-1}$. Combined with Equation 17, the total time complexity can therefore be estimated and is shown as follows

$$T = \frac{T'}{P} \geq \frac{1+4c^2}{2c} \times a^{-1} = O(a^{-1}), \tag{20}$$

which obviously provides an quadratic speedup over the native quantum adiabatic evolution.

We remark that the original success probability defined in Tulsi [14] for the one round of quantum partial adiabatic evolution was given by

$$P' = |\langle E_0(s^-)|\alpha\rangle|^2 \times |\langle E_0(s^+)|\beta\rangle|^2, \tag{21}$$

while in our context here it can be calculated as follows

$$P' = |\langle E_0(s^-)|\alpha\rangle|^4 \approx \left(\frac{1}{2} + \frac{c}{\sqrt{1+4c^2}}\right)^2, \tag{22}$$

From Equations 21, 22, we have used that

$$|\langle E_0(s^-)|\alpha\rangle|^2 = |\langle E_0(s^+)|\beta\rangle|^2. \tag{23}$$

Equation 23 is a symmetry property and easy to verify. As a result, it is easy to check that $P' > P$ for any $c > 0$ by some simple algebraic manipulations, indicating that the per-round success probability defined in Tulsi [14] is overestimated.

3 The quantum partial adiabatic search problem

In this section, we study the quantum search problem using the quantum partial adiabatic evolution framework proposed in the previous section. Suppose we are interested in finding M target elements from a total of N items in an unstructured database. We consider separately the correct and incorrect versions of the quantum partial adiabatic evolution for this problem.

Firstly, for the case exhibiting the optimal quadratic speedup, we do not need to repeat the quantum partial adiabatic evolution procedure, as it directly aligns with our prior discussion. We need only specify that the evolution interval is $[s^-, s^+]$ with $s^{\pm} = \frac{1}{2} \pm c\sqrt{M/N}$. The time complexity, verified using the states $|\alpha\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle$ and $|\beta\rangle = \frac{1}{\sqrt{M}}\sum_{j\in S,|S|=M}|j\rangle$, is $T = O(\sqrt{N/M})$ by Equation 20 with $a = |\langle\alpha|\beta\rangle| = \sqrt{\frac{M}{N}}$. The single-round success probability remains $P \approx \frac{4c^2}{1+4c^2}$ for $M \ll N$.

In several previous works [16, 18, 21, 23], it can be checked that the choices of the evolution intervals are consistent with ours here, and therefore may be considered valid in isolation. Also it leads to a per-round time complexity of $T' = O(\sqrt{N})$ or $T' = O(\sqrt{N/M})$ for $M > 1$, figures that appear consistent even under slightly different problem settings. However, the key flaw identified by Kay Kay [20] concerns the method of calculating the success probability. This error ultimately compromises the overall time complexity analysis in these references, as we will explain.

Next, we turn to the incorrect variant of the quantum partial adiabatic search algorithm, which purports to surpass the established optimality limit of quantum computation. Early works such as those in [15, 17] fall into this category. Our objective is to pinpoint the fundamental flaw in their approach. In these works, the evolution interval was specified as $s^{\pm} = \frac{1}{2} \pm c/\sqrt{N}$ for a search with $M > 1$ targets out of N total items. Consequently, the time complexity T' for a single round of the computation can be directly calculated using Equation 18. Then it follows that $T' \geq O(\sqrt{N}/M)$. Having established this, we proceed to calculate the single-round success probability. By substituting the parameters $s^- = \frac{1}{2} - c/\sqrt{N}$, $a = \sqrt{M/N}$, and $b = \sqrt{1 - M/N}$ into Equation 14 and simplifying, we obtain the following expression:

$$|\langle E_0(s^-)|\alpha\rangle| \approx \sqrt{\frac{1}{2} + \frac{c}{\sqrt{M+4c^2}}} > \frac{1}{\sqrt{2}}, \tag{24}$$

in which we have used that $M \ll N$. The success probability from Equation 16 is thus obtained as follows from Equation 24

$$P \approx \frac{4c^2}{M+4c^2}. \tag{25}$$

This would imply that for fixed constant $c > 0$, the success probability approaches zero as M increases. This is both incorrect and counterintuitive, as we would naturally expect that having more attempts for a larger M should monotonically increase the chance of success. Moreover, when the constant $c > 0$ is chosen sufficiently large but fixed such that P approaches 1, the overall time complexity becomes

$$T = T'/P \geq O\left(\sqrt{N}/M\right), \tag{26}$$

Equation 26 for the quantum adiabatic evolution directly contradicts the proven optimality of $O(\sqrt{N/M})$ for quantum computation. Based on this analysis, we conclude that the choice of the evolution interval $[\frac{1}{2} - \frac{c}{\sqrt{N}}, \frac{1}{2} + \frac{c}{\sqrt{N}}]$ is invalid.

Kay pointed out that the results in the works like [15, 17] were not correct and argued in detail especially why the quantum partial adiabatic search could not achieve an algorithmic performance of $O(\sqrt{N}/M)$ Kay [20]. The root cause of the problem is an insufficient estimate of the algorithm's single-run success probability. This insufficiency, in turn, arises because the overlap between the initial state and the system's ground state was incorrectly bounded by a constant smaller than $1/\sqrt{2}$. However, as shown here, even for the uncorrected quantum partial adiabatic search, this overlap remains greater than $1/\sqrt{2}$. So we have to take a further step to calculate the success probability to see what the actual issue is. Furthermore, Kay proposed that by setting $\delta = c\sqrt{\frac{M}{N-M}}$, the issue identified in the earlier work of Tulsi [14] could be addressed. This parameter choice, which aligns with the interval selection we presented in the previous section, provides additional support for its validity.

Finally, it can be observed that the success probabilities of quantum partial adiabatic evolutions under the two aforementioned circumstances differ. This difference, to some extent, reflects the validity of the quantum partial adiabatic search. Specifically, for the correct version of the quantum adiabatic search, if the constant c is set sufficiently large, the success probability remains close to 1. In contrast, for the incorrect quantum partial adiabatic search algorithm, the success probability decreases monotonically as M increases. Although it can be made arbitrarily close to 1 by adjusting the parameter c, its monotonic decrease with M contradicts our intuition and indicates that the algorithm is flawed.

4 Numerical simulations

In this section, we perform numerical simulations to supplement our analytical results and enhance their credibility. We have conducted two groups of simulations for this purpose, namely, for the valid and the invalid quantum partial adiabatic search.

For the valid quantum partial adiabatic search algorithm, the simulation results are shown as follows. This result examine a complex mathematical function through six complementary

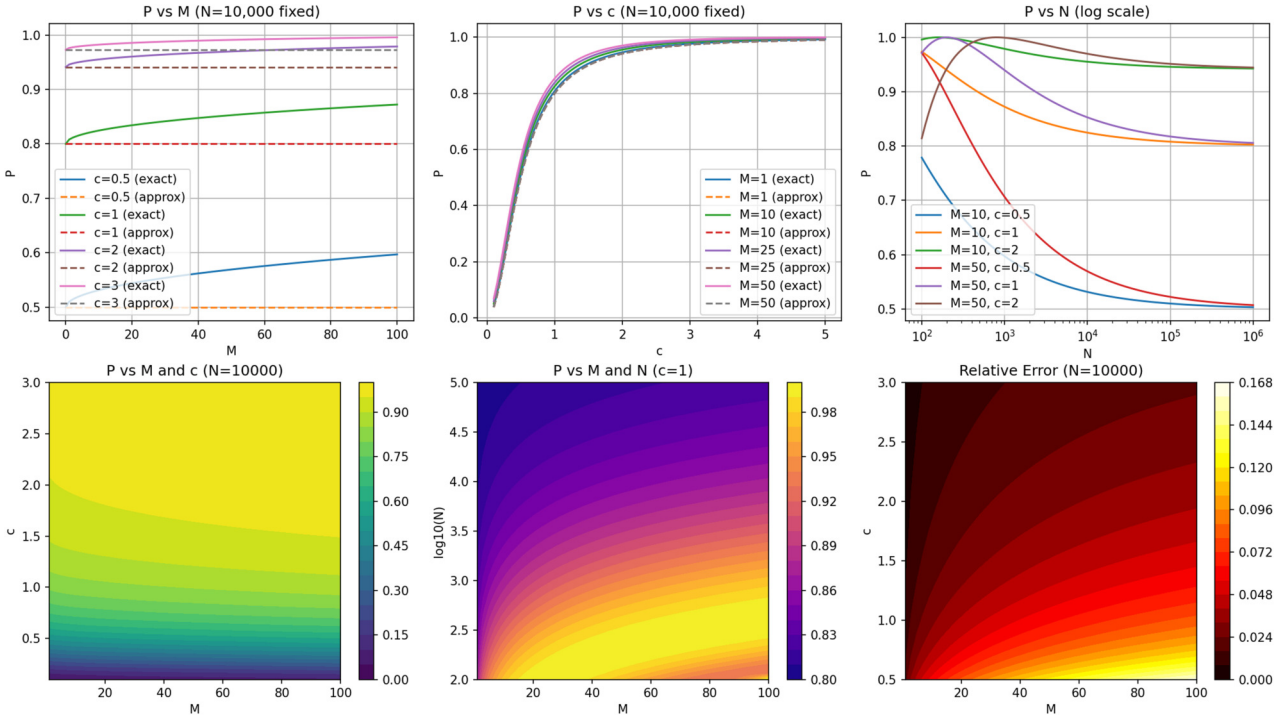

FIGURE 1
Simulation results for valid quantum partial adiabatic search.

visualizations, providing deep insights into the behavior of the analytic expression of the success probability

$$P = \frac{\left[\sqrt{\frac{M}{N}} + 2c\left(1 - \frac{M}{N}\right)\right]^2}{1 + 4c^2\left(1 - \frac{M}{N}\right)} \tag{27}$$

and its relationship with the asymptotic approximation (Equation 25).

The top-left panel of Figure 1 depicts P in Equation 27 as a function of M for a fixed, large value of $N = 10,000$. Multiple curves are shown for different values of $c(0.5, 1, 2, 3)$, each consisting of a solid line (exact solution) and a dashed line (approximation). It can be clearly observed that for any fixed c, P increases smoothly and monotonically with M. In the top-middle panel, we see that for fixed M, P rises rapidly with c eventually saturating near 1. Larger values of M cause the system to saturate at a lower value of c. As shown, the analytic expression and approximated result match so closely for each curve that they are nearly indistinguishable, except when M is large. To examine how P depends on N and to identify the regime in which the large N approximation is valid, we show in the top-right panel a plot of P against N(on a logarithmic scale) for different (M, c) pairs. The results indicate that P is highly sensitive to N only when N is small. As N increases, the value of P for each curve plateaus and approaches a constant. Furthermore, the success probability decreases with increasing N, reflecting the growing difficulty of identifying the marked elements in the quantum partial adiabatic search algorithm. This challenge is mitigated when the target elements are relatively large and the constant c is not too small, as also illustrated in Plot 3.

In the bottom-left and bottom-middle panels of Figure 1, we show two-dimensional visualizations of P as a function of M and c, and of M and N, respectively. Plot 4 synthesizes the relationships from Plots 1 and 2 into a unified representation. The color gradient clearly indicates that high values of P occur in regions where both c and M are large. The function increases smoothly with either M or c. Plot 5 presents a 2D heatmap of P as a function of M and N(on a log_{10} scale) for fixed $c = 1$. As shown, when M is comparable to N(bottom-left region), P is highly sensitive to both parameters, as indicated by the rapid variation in color. In contrast, when $M \ll N$ (top-right region), P depends primarily on M, as evidenced by the vertical banding of colors. In this regime, the value of N becomes less irrelevant, which explains why the approximation performs well here. To quantify the accuracy of the approximate formula relative to the exact calculation across the studied parameter space, we include Plot 6. As shown, the relative error is consistently very low, demonstrating a high level of accuracy over almost the entire range. This provides quantitative evidence of the high quality of the approximation for $N = 10,000$. A slight increase in error is observed for the largest values of M (toward the right edge), which occurs because as M approaches 100, the ratio M/N increases, making the condition $M \ll N$ less strictly satisfied. Nevertheless, the approximation remains excellent across the entire range.

Figure 2 presents the simulation results for the invalid quantum partial adiabatic search algorithm, illustrating the behavior of the analytic success probability

$$P = \frac{\left[1 - \left(1 - \frac{M}{N}\right)\left(1 - \frac{2c}{\sqrt{N}}\right)\right]^2}{1 - \left(1 - \frac{M}{N}\right)\left(1 - \frac{4c^2}{N}\right)} \tag{28}$$

and its relationship with the asymptotic approximation given in Equation 25.

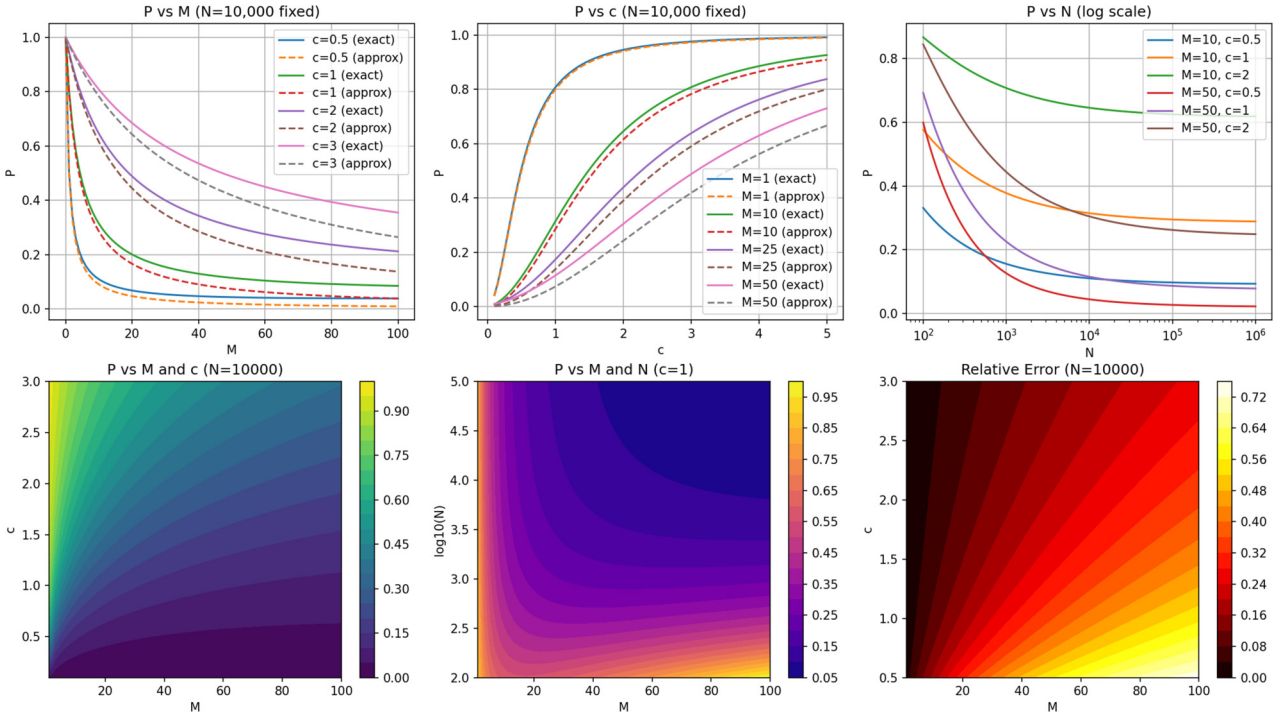

FIGURE 2
Simulation results for invalid quantum partial adiabatic search.

The top-left panel of Figure 2 shows the success probability P in Equation 28 as a function of M for $N = 10,000$, with curves plotted for different values of $c(0.5, 1, 2, 3)$. Each curve is presented in two forms: the exact solution (solid line) and the approximate solution (dashed line). As seen in Plot 1, for any fixed c, the value of P monotonically decreases with M, which contrasts with the behavior observed in Figure 1. The approximation becomes increasingly inaccurate as M grows, consistent with the assumption $M \ll N$ used to derive the simplified expression for P. In the top-middle panel, we observe that for fixed M, P increases rapidly with c, but only for small M does it saturate near 1. This reveals a counterintuitive feature of the quantum partial adiabatic search algorithm: a larger number of target elements does not necessarily facilitate the search process. Furthermore, in sharp contrast to the behavior in Figure 1, we observe that the top-middle panel of Figure 2 shows that the exact and approximate results for each curve are in close agreement only for $M = 1$, with a clear discrepancy for all other cases. The top-right panel (Plot 3) examines the dependence of P on N (on a logarithmic scale) for different (M, c) pairs. It confirms that P is sensitive to N only when N is small. As N increases, each curve flattens and approaches a constant value. The decrease in success probability with larger N is intuitive, reflecting the increased difficulty of locating marked elements in a larger search space. However, this difficulty is not mitigated by having more target elements, as larger M still results in lower P, as seen in the plot.

In the bottom-left and bottom-middle panels of Figure 2, we present two-dimensional visualizations of P as a function of M and c, and of M and N, respectively. Plot 4 integrates the trends from Plots 1 and 2 into a single comprehensive view. The color gradient clearly indicates that high values of P are concentrated in regions with high c and low M. The function P decreases gradually as M increases or as c decreases. Plot 5 shows a 2D heatmap of P as a function of M and N(on a log_{10} scale) for fixed $c = 1$. When M is comparable to N (bottom-left region), P remains highly sensitive to both parameters, as indicated by the sharp color variations. In the top-right region, where M is large and N is fixed at a high value, P becomes extremely small, which is consistent with the expression given in Equation 25. Finally, Plot 6 quantifies the accuracy of the approximate formula across the studied parameter space. The relative error remains low only when M is small and c is large. As M increases or c decreases, the approximation deteriorates. The rise in error for large M is expected: as M approaches 100, the ratio M/N increases, making the condition $M \ll N$ less strictly satisfied. Nevertheless, the overall behavior of the approximation remains consistent and interpretable.

In summary, Figures 1, 2 clearly differentiate the valid and invalid quantum partial adiabatic search algorithms by their distinct dynamic behaviors.

5 Conclusions and discussions

In this paper, we propose a framework for quantum partial adiabatic evolution and apply it to the quantum search problem. Our main findings are summarized as follows. As can be seen, our setting here is simple enough to analyze compared with that of Tulsi [14]. For a valid quantum partial adiabatic search, which means that its time complexity matches the established optimality of quantum computation, the evolution interval must be chosen as

$[\frac{1}{2} - ca, \frac{1}{2} + ca]$. Here, $c > 0$ is a constant and a is the overlap between the initial and final states. Furthermore, we show that the success probability of a single round of adiabatic evolution can be made arbitrarily close to 1 by selecting a sufficiently large value of c.

On the other hand, the so-called "improved" quantum partial adiabatic search, which claims to achieve a performance beyond the standard quadratic speedup, such as $O(\sqrt{N}/M)$, is in fact incorrect. This judgment holds even when the evolution interval is specified as $[\frac{1}{2} - c/\sqrt{N}, \frac{1}{2} + c/\sqrt{N}]$ and the overlap between the initial state and the ground state at s^- still satisfies the constraint, i.e., being greater than $1/\sqrt{2}$ outlined in Kay [20]. Furthermore, it is observed that the success probability can become arbitrarily small as the number of target elements increases, a result that clearly contradicts intuitive expectations. The result on the invalidity of the quantum partial adiabatic evolution here, is corroborated by prior research. The findings of Sun et al. [24] and the optimality proof in Mei et al. [23] collectively imply that any attempt to exceed the fundamental quadratic speedup of quantum over classical computation cannot succeed in the circumstance of quantum partial adiabatic search.

Our findings provide a clear framework for re-evaluating the literature on quantum partial adiabatic computations. We identify two distinct types of flaws in prior works. The first type, exemplified by studies such as Zhang et al. [16]; Sun et al. [18, 21], Sun and Lu [25], stems from an incorrect method for calculating the success probability. While their choice of evolution interval is itself valid, their analytical approach to estimating the probability of success within that interval is flawed, and our results offer a direct corrective. The second, more fundamental type of flaw, as also noted by Kay [20] and evident in works like Zhang and Lu [15]; Sun et al. [17], concerns the choice of the evolution interval itself. Our results unequivocally demonstrate that their selected intervals are incorrect, as they do not satisfy the theoretical prerequisites for achieving a high success probability. Additionally, our analysis is further confirmed by numerical simulations, which show a clear distinction between the valid and invalid quantum partial adiabatic search algorithms.

Our work complements recent efforts to establish criteria for valid partial adiabatic search, including those in related studies Sun et al. [22], Sun and Zheng [26]. We hope our results will contribute to a deeper understanding of the quantum partial adiabatic evolution paradigm, which, despite its potential, remains less explored compared to other quantum adiabatic computing approaches.

The implications of our framework extend beyond the specific model studied here. A promising future direction is its application to more general quantum optimization problems, such as combinatorial optimization tasks encoded in Hamiltonian-based formulations. In this context, our method could offer a refined strategy for setting partial adiabatic annealing schedules, potentially leading to performance improvements. Furthermore, within quantum machine learning, this framework might be adapted to analyze the training dynamics of parameterized quantum circuits, possibly providing insights into mitigating barren plateaus by ensuring more controlled evolution through the parameter landscape.

However, several important limitations must be addressed for practical applications. As we consider scaling to high-dimensional systems, the interplay between the density of states and the minimum gap becomes more complex; our current analysis, which may rely on specific spectral properties, would need generalization to handle highly degenerate or chaotic energy spectra. Moreover, the framework's robustness against environmental noise and decoherence is a critical open question. In real-world, open-system conditions, the adiabatic condition must be satisfied within finite coherence times. Future work should integrate techniques from open quantum systems, such as the adiabatic master equation, to quantify the trade-offs between evolution speed, system size, and noise resilience, a crucial step for deploying such frameworks on current noisy intermediate-scale quantum (NISQ) devices.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

JS: Writing – review and editing, Writing – original draft. HZ: Writing – review and editing. SL: Writing – review and editing, Validation.

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Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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