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Foreground and background: fractal unity of art and science

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Introduction: In the last 50 years, fractals have been embraced by both artists and scientists. In this study, we link fractals to the question: What is the relationship between art and science? Clearly, both results of scientific experiments and expressions made by artworks contain mixed truth and falsity. Reflection shows that, in artistic expression, the public experiences falsity (the artifice) in the foreground and discovers truth in the background, while in science, the opposite applies. This is captured by the dictum "Art is the lie that tells the truth/science the truth that lies." Truth behind lies or lies behind truth are revealed as one examines deeper and deeper levels. We modeled this progressive intermingling of truth and falsity in art and science with a fractal metaphor. In the metaphor, falsity (the artist's or scientist's subjective input) is the fractal frame (its interdigitated void spaces), while truth (Nature's objective response) is the fractal fill (its positive spaces).
Methods: Using a simple fractal (the Cantor Dust), we worked out an elementary formalization of this concept of "fractal truth" on an example problem (shallow-bed filtration). To assess possible presence of fractal character within art and science, we cast two hypotheses. H1: Both art and science exhibit one or more fractal properties. H2: Art and science manifest these properties in contrasting foreground/background manner. We tested H1–H2 using a formal comparative analysis. Thus, we identified five general properties of fractals: dimensions, infinity/the infinitesimal, novelty/familiarity, ellipsis (gaps in representation), and the personal (role of the individual fractal analyst, artist, or scientist), illustrating each with example fractals (e.g., Koch snowflake, Sierpiński triangle, Pascal's triangle modulo 5, Mandelbrot Set, and Weierstraß Function).
Results: Comparing art and science, we found that all five properties were present in both disciplines and that all five manifested in a contrasting foreground/background manner in art vs. science.
Discussion: We compare this fractal picture with prior notions of truth in art and science and suggest possible applications. We conclude that a fractal perspective might promote a more unified and harmonious understanding of art and science on this Golden Fractal Anniversary.

KEYWORDS
fractals, art, science, truth, dimensions, infinity, novelty, ellipsis

1 Introduction

Mandelbrot [1] demonstrated the ubiquity of fractals in Nature. Their utility and fascination for science have grown greatly since [2–6]. In art, fractals trace back centuries but have exploded in the past 50 years [7, 8]. In art and science, we inhabit a Golden Age of Fractals. This paper relates fractals to the research question: What is the epistemological relationship between art and science?

The capacity of art and science to convey truth and the type of truth(s) sought by each has been debated since Antiquity and remains a focus today [9–16]. In the following sections, we argue the relationship of art and science to truth is contrasting and complementary. The shared embrace of fractals by both disciplines hints that fractals may help discern their differences and similarities. In this study, we investigate this possibility.

In the Methods section below, we observe that every scientific experiment is an interaction between the scientist and the sample, and every artwork is an interaction between the artist and the medium. The results of the experiment or the expression afforded by the artwork always contain mixed truth (Nature's input) and falsity (artifice and human input). In art, the public experiences falsity in the foreground and discovers truth in the background; in science, truth is in the foreground, falsity in the background. This is captured by the dictum "Art is the lie that tells the truth/science is the truth that lies," our leitmotif. The truth behind the lie or the lie behind the truth is revealed as one examines a work of art or science at deeper and deeper levels. Such intermingling of truth and falsity recalls how fill and void interdigitate down to ever-finer resolutions in fractals. Since our proposed relationship between art and science concerns the connection of each to truth, we developed a *fractal metaphor* for truth in art and science. This "fractal truth" concept is a heuristic analogy intended to portray overlap and difference between art and science. In the metaphor, which makes no metaphysical claims, truth (Nature's objective input) is the fill, the fractal positive space, and falsity (the artist's or scientist's subjective input) is the negative void space framing the fractal. The enterprise of art is to construct frames, and that of science is to unveil fills. To the extent art lives within science, it includes the hypotheses and interventions scientists impose upon Nature in experiments; to the extent science inhabits art, it includes impartial observations and oppositional contacts artists makes with material reality through tools and media. Art and science are thus essentially the same enterprise, only with a difference in emphasis: foreground vs. background. We explain the concept of "fractal truth" below and formalize it with an elementary example.

To assess possible fractal character within art and science, we cast two hypotheses: H1: Both art and science exhibit one or more fractal properties, and H2: Art and science manifest these fractal properties in a contrasting foreground vs. background manner. We test H1 and H2 using a *formal comparative analysis*. Thus, we identify five general properties of fractals: role of dimensions, tendency toward infinity and the infinitesimal, novelty and familiarity, use of ellipsis (exclusion in expression), and role of the *personal* (arbitrary choices of the individual). Illustrative fractals include the Koch snowflake, Sierpiński triangle, Pascal's triangle modulo 5, Mandelbrot set, Weierstraß function, and coast of Britain. In the Results section, we determine whether each property appears in each discipline and whether it manifests in foreground/background opposition in art vs. science.

2 Methods

2.1 How art and science relate to truth

In this section, we explain our leitmotif: "Art is the lie that tells the truth/science the truth that lies." The first half translates Picasso's [17] famous quote, «Todos sabemos que el arte no es verdad. El arte es una mentira que no acerca a la verdad, al menos, a aquella verdad que se nos da para entendar»; we added the second half. Together, the two propose a complementary symmetric relationship between the two disciplines. Before we explain, a disclaimer: This thesis is *not* about fraud. Fraud is a perennial, but second-order, issue in art and science, and not our topic. Instead, we seek the primary relationship of art and science to truth, regardless of honest or deceptive motives.

"Art is the lie that tells the truth" is explained by [18–20]. Imagine you are beholding a work of art …theatre …Beckett …Krapp's Last Tape (Figure 1 [21]). You know the man on stage is not really Krapp. He is an actor. He is not recounting his own life but the life of Krapp, a fictional character. It is all an *illusion*. Yet, it conveys profound—and disturbing—truths about life and humanity. It does this because the illusion triggers familiar notions in the audience: growing up in a family …seeking a mate …struggling in a career …and contending with mortality. The truths are in the minds of the public. It is an *objective* illusion and a *subjective* truth. The same applies to any artwork. In this way, art is self-evidently the lie that tells the truth.

"Science is the truth that lies"? This is less evident. We consider it in detail with an example from brain research. Imagine you are at a talk …psychiatric neuroimaging …a review …. A distinguished professor speaks. Arm upraised, he motions at a slide (Figure 1 [22]), proclaiming "This …is …the …neuronal …activation …of a subject's …dorso-lateral pre-frontal …cortex …during …an act of …motor …language!" Only it is not *really*.

The slide shows a *structural MRI*, a picture of the brain. However, the real brain is different. Its cortex is pink, not gray. It is not still; it pulsates. The yellow-red blob atop the MRI represents a functional MRI (fMRI) result. It supposedly denotes brain regions with "neuronal activation" via the blood oxygen-level-dependent (BOLD) effect. However, BOLD does not depict neuronal activation in isolation but rather in combination with blood flow. Furthermore, BOLD has a time delay (here 4–8 s); it was not measured "during" but "after" the action. In addition, as with most fMRI, the signal had to be averaged across subjects to detect a visible effect. Thus, the picture does not show the fMRI for a single human being but for a group. Nor are the blobs the fMRI scan itself; actual BOLD scans rarely appear in papers. The blobs are statistical parametric maps and so forth. However, most in the audience know at least some of this. They know, more or less, that many unmentioned preparative steps went into the images. It is all an illusion. However, the lecture works because the public is fully or at least momentarily unaware of the illusions or notices but expediently opts to accept them. This example illustrates that a scientific study (here, neuroimaging) is every bit as contrived as a stage play. Again, it is a subjective truth and an objective illusion. Yet, our example is from an excellent study [22]

FIGURE 1
Art is the lie that tells the truth; science is the truth that lies. **(A)**John Hurt in Beckett's Krapp's Last Tape, Kirk Douglas Theatre, Los Angeles, October 2012 (Craig Schwartz Photos, used with permission). **(B)**Axial-oblique section of T1-weighted MRI of a human brain with superposed results of functional magnetic resonance imaging (fMRI; *yellow blob*) scans acquired during a verbal fluency task ([22]; used with permission).

that yielded highly plausible and worthwhile findings. In this way, science is the truth that lies.

2.2 Foreground and background in art and science

This section outlines the foreground/background contrast between art and science. Table 1 shows this relationship arising in the design, conduct, and appraisal of a work of art or science. It provides scope to the fractal truth concept. To evaluate fractal truth (or falsity) *in science*, the *public* (general public, colleagues, reviewers, and even the investigators themselves) examines the *results* of an *experiment*. Results are examined at the end of the experiment or as they emerge. An evaluation of fractally true or fractally false is made for hypotheses cast beforehand, for validating statements that operationalize the hypotheses, or even for individual manipulations that non-verbally probe the sample (and thereby advance or impede hypotheses). The values are assigned on successive levels of a procedural network that continues to an ever-finer scale. If a contention (hypothesis or validation statement) is "fractally true," that means Nature accords with it *thus far* within the network; if the contention is "fractally false," Nature broke with it at some point. Fractal modeling maps out the outcome profile of procedures, validates them down to a certain level, probes for emergent issues at deeper levels, and selects auspicious investigative strategies. The Hausdorff dimension and other parameters of the modeling fractal provide metrics for the complexity of the network.

To evaluate fractal truth *in art*, the public (general public, colleagues, critics, and the artist) examines the *expression* (the realized ensemble of potential effects) of an *artwork*. Expressions are examined during the creation of the piece as they emerge or after it is complete. Fractal truth or falsity is evaluated for *design motifs* (intended effects and implanted ideas) chosen before (possibly during) production, schemata (e.g., drafts) of the motifs, or non-verbal manipulations acting on the piece (advancing or impeding a motif). Values are assigned within a multi-level procedural network. There is a twist here. We keep the definitions of truth and falsity *the same* in art and science. Therefore, in art, a motif, schema, or manipulation is fractally "false" when it conveys the artist's input and fractally "true" when it retains Nature's native input, both within the network (one could, alternatively, reverse these and speak of "separate truths" in art and science, as many do, but we choose not this path). Fractal modeling maps out possible outcomes of this process of design, creation, and appraisal. It validates procedures up to a certain level, detects possible issues at deeper levels, and helps choose fruitful creative strategies. The Hausdorff dimension and other fractal parameters again characterize the complexity of the process. Thus, we have the scope of fractal truth in art and science.

In step 1 of Table 1 (left), an artist chooses one or more design motifs, thematic elements to incorporate into the artwork. In step 2, the artist creates the piece. This entails numerous manipulations, e.g., brush strokes, saw cuts, stainings, or any intervention intended to infuse design motifs into the medium. In step 3, as the artwork is completed, it projects or emanates a certain artistic expression, e.g., bears an image, assumes a form, strikes a tone, or any of infinitely many artistic end-products. In step 4, the end-product combines the input of Nature with that of the artist. The former, in the fractal metaphor, is truth (T), the fractal fill; the latter is falsity (F), the void space that frames the fill. The design intent is to embody the artist's input in the expression. In step 5, the public examines the expression, e.g., views the painting or statue, hears the concert, or otherwise appreciates the artwork. This creates an impression on the public (step 6). If the expression is effective, in step 7, the public experiences the artist's input mainly in the foreground, Nature's input mainly in the background. For example, the spectator sees Krapp, Whistler's mother, or the ass who bore Christ into Jerusalem—first—and John Hurt, oil smeared on canvas, or a marble block—second. Having bought-in to the illusion, however, the public may yet be led to deeper superseding truths, the artist's spoken or unspoken higher purpose.

In step 1 of Table 1 (right), a scientist casts one or more hypotheses to be tested experimentally. In step 2, the scientist conducts the experiment. This entails numerous manipulations, e.g., injecting solutions, X-ray scans, heating with Bunsen burners, or any intervention that challenges the sample with the hypotheses.

TABLE 1 Fractal conception of truth and falsity in art and science.

Step	Art	Science
1	Artist chooses design motifs	Scientist casts hypotheses
	↓	↓
2	Artist creates artworks Manipulations enact design motifs ↓	Scientist conducts experiments Manipulations enact hypotheses ↓
3	Artworks produce expressions	Experiments yields results
	↓	↓
4	Expressions contain T + F T is the fractal fill T is the Nature's input T is objective F is the fractal void (frame) F is the artist's input F is subjective (artist's intent) ↓	Results contain T + F T is the fractal fill T is the Nature's input T is objective (scientist's intent) F is the fractal void (frame) F is the scientist's input F is subjective ↓
5	Public examines expressions	Public examines results
	↓	↓
6	Public has impression	Public has impression
	↓	↓
7	Public experiences F in the foreground Public discovers T in the background	Public experiences T in the foreground Public discovers F in the background
	"Art is the lie that tells the truth"	"Science is the truth that lies"

T, truth; F, falsity.

In step 3, the experiment yields results, e.g., chart-recorder traces, radiographs, thermometer readings, or any of infinitely many scientific end-products, often summarized in tables or graphs. In step 4, we note that the end-product combines Nature's input with the scientist's input. Nature's input is T, the fill of the fractal; the scientist's is F, the frame. The scientist's intent is to record Nature's input as purely as possible. In step 5, the public examines the results, e.g., inspects the tables or graphs. This creates an impression on the public (step 6). If the results are effective, in step 7, the public experiences Nature's input mainly in the foreground and the scientist's input mainly in the background. Having accepted the foreground results as facts, however, the public may yet be deceived if results are overturned upon deeper investigation, a spoken or unspoken proviso of the scientist.

Table 1 implies that, at base, art and science are the same enterprise. The difference is in emphasis. Both yield subjective truth and objective illusion. In art, however, artifice is in the foreground and truth is in the background, whereas in science, the opposite pertains. With a work of art, we know upfront that the artist aims to deceive. A still, 2D portrait or landscape stands for a moving, 3D person or terrain. We are eager to feel how convincingly the artist takes us in—or to protest if the veil pierces too easily. Science, we presume upfront, is supposed to tell the truth. Our attitude is skeptical; we are angry when deceived. Yet, even in science, we are taken in unconsciously by artifices. As observed in the psychiatric neuroimaging above, *a lot* is under the hood. Moreover, the hidden parts are important. One well-designed study [23], for example, provided the same neuroimaging dataset to 10 different research laboratories to process. Agreement in outcome was under 30%. Many arbitrary intermediate processing steps changed the final results. In both science and art, illusions are pervasive. However, fortunately, they are surmountable as science and art march on.

Artists will recognize our argument from Magritte's celebrated painting *La Trahison des images* (*The Treachery of Images*). A fuming tobacco pipe spans a canvas above the inscription «Ceci n'est pas une pipe» ("This is not a pipe"). The viewer is perplexed, perhaps indefinitely, until realizing, "Oh, it is not a *pipe*; it is a *picture* of a pipe!" The work embodies a paradox at the core of representational art. Science also has paradoxes. One is the quantum Zeno effect, which we have discussed in relation to psychiatry [24]. Even mathematics has paradoxes. We proposed, for example, a paradoxical definition of natural numbers [25]: "A number is a universal symbol that treats different things as alike and (simultaneously) alike things as different." In other words, in counting, one groups clearly distinct objects together despite their differences; alternatively, one labels substantially identical objects

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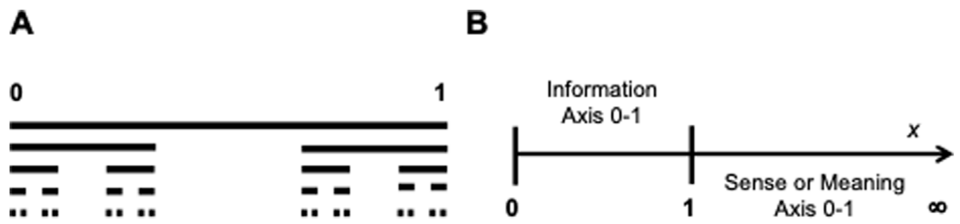

FIGURE 2
Cantor dust, an elementary fractal; elements of a dimension. **(A)** To construct a Cantor dust, remove the middle third of a line segment of length 1 (*uppermost bar*). Then, repeat the same for each remaining segment (*shorter*, *lower bars*) *ad infinitum*. The result is an infinite series of points with constantly remaining interstitial empty space. The sketching of multiple parallel bars is an artificial convention, and the actual Cantor set consists of a single line of more and more finely distributed points (modified from [26].). **(B)** *Dimension* (degree-of-freedom, coordinate axis, or attribute [25]). Art and science routinely measure and create effects along an endless number of novel and familiar dimensions. A dimension requires three things, all observed on the axis for attribute *x*: a place to start (a 0), a direction to proceed (an infinity or maximum), and the size of steps to take along the way (a 1). In art and science, dimensions are integrated into a work with attention to which aspects come to consciousness and which remain hidden (Modified from [25]).

as distinct. However, in accepting the artistic paradox, we win representational art; in accepting the scientific paradox, we win quantum mechanics; and in accepting the counting paradox, we win realms of mathematics. Although paradoxes are typically used to invalidate arguments, a few central paradoxes are generative, as when negating Euclid's Fifth Postulate yields entire branches of non-Euclidean geometry. Thus, if "Art is that lie that tells the truth" and "Science is the truth that lies" are paradoxes, they may nonetheless foster understanding.

2.3 Fractals and truth in art and science

2.3.1 Cantor dust: structure and properties of an elementary fractal

To explain the proposed fractal relationship of science and art to truth and falsity, we first review the (triadic) *Cantor dust*, one of the simplest fractals (Figure 2). As readers know, to generate a Cantor dust, start with a horizontal bar. Remove its middle third. Then, remove the middle third of each of the two remaining shorter bars. Continuing the process *ad infinitum,* we obtain shorter and shorter bars with more and more spaces in between, but we never lose the bars entirely. In the process, we produce a Cantor dust. The Cantor dust, like all fractals, is "everywhere sparse." It is present everywhere and absent everywhere (a generative paradox lies at the heart of fractals). Since the naked eye cannot see down to the lowest level, in drawing the dust we *stop* at some point. The dust has the celebrated fractal property of *self-similarity* or *scale-invariance*. No matter how deep you go inside, you can always re-find the *generator* of the fractal—here, the solid bar. However, the dust looks *different* at each successive level; the gaps and fills become shorter as the resolution increases. Whenever we look closer, we must revise our concept of the absolute structure of the Dust.

2.3.2 Motivating fractal truth for art and science

Much like the Cantor dust, in our conception, truth is everywhere sparse. We have a frame and a fill. The frame is artificial, and the fill is natural. The frame arises from the artist's or scientist's interventions and efforts to impose a hypothesis, and the fill is Nature's response. When challenged by the artist, Nature may assume the desired form, assume the opposite form, or do nothing; when challenged by the scientist, Nature may falsify the hypothesis, not disprove it, or answer indeterminately. We enjoy the artificial beauty of the frames and the natural beauty of the fill. Together, frame and fill build the interdigitated fractal-like reality we confront in art and science.

Some examples of truth in science are provided as follows. In genetics, early on, we learn that a *species* has a fixed number of chromosomes—46 for humans. Later, we encounter people with trisomy X, mongolism, YYY syndrome, and diverse conditions with *other* than 46 chromosomes. These people are clearly human, so closer familiarity compels us to revise our concept of species to cast a new frame. Another example: early on, we learn that each human has a specific personal genome, serving as a genetic identity in every cell of the body. Digging deeper, we learn the genome is only in *nucleated* cells and absent in anucleated erythrocytes and platelets. Deeper still, we find, even for nucleated cells, that the genome varies by *tissue* within the selfsame human. Like fractals, we revise as we go deeper. This applies to both the learning of individual students and the progress of the field. The closer we examine Nature, the more we revise our beliefs. Eventually, we wonder what scientific statements can we retain at all?

Yet, truth is ubiquitous. We ignore it at our peril. Drifting off into a daydream on a Sunday walk, you are recalled to pitiless reality upon stubbing your toe. Scientists face this daily. Say, e.g., you are assembling electronics to record an EEG. You need a male connector. Here is one. No! That is a Berg; you need a Molex connector. Here. No, wrong size. OK, this one fits—insert it. No, it will not go in. Each channel of the female is blocked with a plastic chad. You must break them open. With this screwdriver? No, too fat. A low-gauge syringe? It clears an opening, but leaves fringe. That will suffice—it works. All for one piece of apparatus for one run of one experiment in one project that may or may not succeed. Welcome to a typical day in the laboratory! Objective truth repeatedly confronts and evades the scientist at finer and finer levels. Truth is everywhere dense. [27], therefore, emphasized the importance of doubt in science, of uncertainties that emerge and are corrected at finer and finer levels, but never ultimately resolve. The philosopher Kierkegaard [28] offers contrasting wisdom, speaking of the "monster of doubt." If you have only doubt, you have no relief and no power. Since there is always residual doubt, it is essential for scientists, artists, and everyone to act in faith at some point, to take chances, to treat something as true without evidence, and to pretend something is untrue though we know it is. In drawing Figure 2, at some point, we must stop dividing the fractal bar into ever finer segments. Doubt is in the foreground in science with its normative skepticism and in the background in art. In art, faith is in the foreground; in science, it is in the background.

In seeking genuine representation or alternative aesthetic goals, the artist, like the scientist, corrects repeatedly at finer and finer detail, finally terminating at some arbitrary cut-off. Leonardo, for example, one of history's greatest artists, left many works incomplete. It is said he did this because he conceived in his mind subtleties too fine to resolve in actuality. The struggle with material reality is a commonality of art and science. Neither the scientist in the laboratory with the sample and instruments nor the artist in the studio with tools and media gets past material objectivity at any instant in any aspect. Nature has a mind of her own, not subject to our whims, only to our concrete efforts to shape her. Thus, truth, like fractals, is "everywhere sparse." This epistemic architecture of reality may condemn art to be the lie that tells the truth and science to be the truth that lies.

2.3.3 Contextualizing and positioning fractal truth

We have not encountered the concept of fractal truth used in our model elsewhere. Thus, we need to contextualize this term and position it within the complex topic of truth as an absolute property. Several theories of truth have emerged since antiquity. These include the classical correspondence theory [29], the coherence theory [30], Tarski's [31] semantic theory, and the constructivist theory [32]. Science is undoubtedly a highly pragmatic discipline. Art is, arguably, also pragmatic. The artist is always seeking something that "works" to achieve an aesthetic or practical effect or another goal. Appropriately, fractal truth as employed here fits best with *pragmatic theories* of truth [33]. Thereby, it retains elements of Tarski [31]. Fractal truth concerns not absolute truth but truth in practice. Science is an effort to establish truth. Art is an attempt to preserve, express, and capture it in fact and feeling. Our fractal model aspires to a view of art and science that confronts this. The fractal truth concept encompasses evaluating scientific hypotheses and other contentions, whether stated explicitly in theorizing or presenting results or enacted as individual manipulations in conducting or preparing experiments. Likewise, the scope of fractal truth includes evaluating artistic sentiments and impressions whether stated explicitly in criticism or presentation or enacted as individual manipulations in preparing works of art. The evaluation is made by comparing the statement with prior or future experimental findings or examining the results of an individual manipulation. Likewise, in art, one evaluates whether—and how well—a particular intended (or plausibly intended) aesthetic, emotional, or practical effect was achieved by the work or manipulation. Or one evaluates statements made by or about a work of art.

2.3.4 Elementary formalization of fractal truth

In Figure 3, a shallow-bed filtration problem demonstrates an elementary formalization of the fractal truth heuristic. The solution follows the scheme of the Cantor dust (Figure 2). The series of experiments in the protocol corresponds to successive levels of resolution in viewing the dust. Thereby, the roles of fill and void are inverted whenever suitable. Other problems could use other fractals. The endpoint need not be length; it can be any measurement variable. We quickly mention that the method portrayed is illustrative—not what one would do in practice. For example, some maneuvers could be combined, but here we work in microsteps.

The problem of Figure 3 concerns particles of species A and B co-suspended in a liquid. The goal is to determine the particle-size distributions (the largest and smallest particle diameters) of both species by running the suspension through Millipore filters. We assay the particle content of the effluent and the sediment cake at the filter influent. The fractal solution procedure entails casting a series of hypotheses. We formulate a *validating statement* to operationalize each hypothesis. We then run an experiment. The experiment evaluates whether the statement is true (T) or false (F) for each value of a relevant parameter (e.g., pore size). All possible outcomes are plotted schematically. Logic is applied to the results to calculate endpoints as opportunities arise. Based on the outcomes, we achieve a solution, reach a dead-end, or repeat the procedure with the next hypothesis.

The solution to Figure 3 starts with Experiment 0 (E0; baseline). We simply gather filters across a range of pore sizes $D^{mn} - D^{mx}$ (e.g., 0.2–20 μm). After E0, once we have our filters, we can pose Hypothesis 0 (H0): $D^{mn} \le D_B, D_A \le D^{mx}$, i.e., the available pore sizes span the particle sizes of A (D_A) and B (D_B). Therefore, in principle, we can separate them. H0 is operationalized by statement S0: "The filter blocks all A and B or passes all A and B." Three comments on S0 (and succeeding statements S1, S2, …): 1) S0 charts a *via negativa*; it is constructed to rule-out undesired outcomes before homing in on promising outcomes; 2) S0 is equivalent to a question (Q0): "Is the filter useless?"; 3) S0 is a logical couplet (an OR statement). One could evaluate the two halves of S0 separately, but we find that coupled statements reveal the covert fractal structure of this procedure of empirical inquiry. S0 is evaluated by E1. In E1, we run a sample of the suspension through each filter across $D^{mn} - D^{mx}$. In Figure 3, we plot the possible outcomes of E1 as three line segments, each spanning the interval $D^{mn} - D^{mx}$. One outcome (left) is that S0 is T for the entire range. In other words, every filter is useless; it either blocks everything or passes everything. We need larger or smaller pore-sizes. H0 is F. Another outcome (right) is that S0 is F everywhere. The particle sizes span the pore sizes, rather than *vice versa*. We need a broader range of filters. H0 is again F. The middle outcome is the interesting one. Like the triad of the first level of the Cantor dust, it divides the interval into three segments (though not necessarily of equal length). S0 is T for the first and third segments and F for the second. This implies H0 is T; we have a useful filter range. Moreover, logic tells us D_B^{min} and D_A^{max} are the lower and upper cut-offs for the middle segment. So, assuming we obtain the middle outcome, we are headed toward a particle size distribution.

We now cast H1: $D_B^{min} \le D_B \le D_B^{max} < D_A^{min} \le D_A \le D_A^{max}$, i.e., A and B have separate (non-overlapping) particle size distributions. To test H1, we formulate S1: "The influent cake contains mixed A and B, or the effluent contains mixed A and B." H1 focuses on the shorter interval $D_B^{min} - D_A^{max}$ (blown-up in Figure 3) internal to $D^{mn} - D^{mx}$, like inspecting a fractal at higher resolution. E2 is conducted on this interval. In E2, we run the suspension through each filter in this segment, examine the outputs, and plot the results (in reality, the

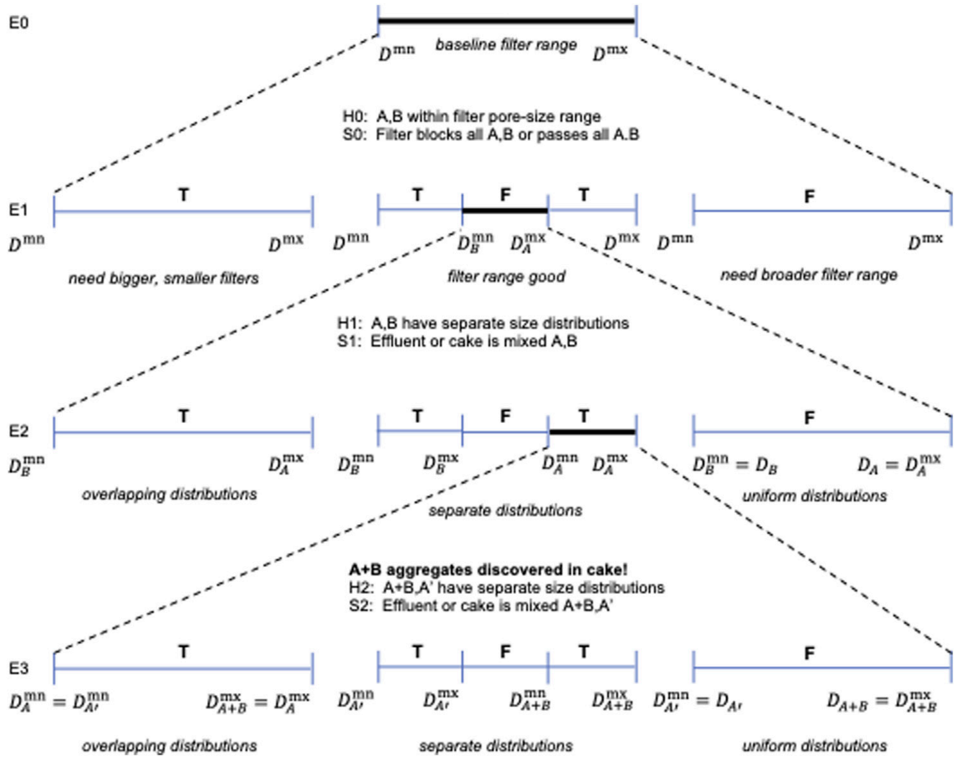

FIGURE 3
A shallow-bed filtration problem demonstrates an elementary formalization of the fractal truth heuristic. See text for details.

"experiment" may consist of merely reanalyzing the filtration data from E1 under the new hypothesis). The first possible outcome (left) is that S1 is T across the entire interval. Every filter yields mixed output. H1 is F, and we have *overlapping* distributions. Another outcome (right) is that S1 is F everywhere on the interval. This is the happy case that every filter produces only A in the cake and only B in the effluent (and 100% yield separation). H1 is T, and we have *uniform* distributions (all particles are the same size) for both A and B. The particle sizes, moreover, are $D_B = D_B^{\mathrm{mn}}$ and $D_A = D_A^{\mathrm{mx}}$. Therefore, we are done. For the middle outcome, the segment is again broken-up into a triad, like the second level of the Cantor dust. S1 is T for the first and third segments but F for the middle segment. This again implies that H1 is T (and not just as a limiting case) and that A and B have separate particle size distributions. Moreover, the lower and upper cut-offs of the middle segment are D_B^{mx} and D_A^{mn}, respectively. Again, with this outcome, we are done; it fully characterizes the system.

Suppose, however, we reexamine the influent cake and discover that, while the smaller of the alleged A particles are genuinely pure A, the larger particles are actually *aggregates* of A + B pairs. Unfortunate, but for certain samples, the fractal procedure still solves the problem. We label the pure A particles A' (retaining the label "A" for the mixtures of A′ and A + B observed in E1 and E2). We cast a new hypothesis, H2: $D_A^{\min} \le D_{A'} \le D_{A'}^{\max} < D_{A+B}^{\min} \le D_{A+B} \le D_A^{\max}$. This hypothesis states that the pure particles A′ and the A + B aggregates have separate distributions. We test it with S2: "The influent cake contains mixed A′ and A + B, or the effluent contains mixed A′ and A + B." In E3, we scrape the A cake off a filter in the $D_B^{\mathrm{mx}} - D_A^{\mathrm{mn}}$ interval of E2 since it should contain no free B particles. We re-suspend the cake and run it through filters in the adjacent interval $D_A^{\mathrm{mn}} - D_A^{\mathrm{mx}}$. In the first outcome (left), S2 is T everywhere in $D_A^{\mathrm{mn}} - D_A^{\mathrm{mx}}$. This implies overlapping distributions of A′ and A + B, and H2 is F. In the third outcome (right), S2 is F everywhere; H2 is T, and we have uniform distributions of A′ and A + B. In this case, $D_{A'}^{\mathrm{mn}} = D_A^{\mathrm{mn}} = D_A$ and $D_{A+B}^{\mathrm{mx}} = D_A^{\mathrm{mx}} = D_{A+B}$ for all A′ and all A + B particles, respectively. In the middle case, the interval is again split into a triad of smaller segments, like the third level of the Cantor dust. S2 is T for the left and right segments and F for the middle segment. H2 is T, and we have separate distributions of A′ and A + B. The cut-offs of the middle segment are $D_{A'}^{\mathrm{mx}}$ and D_{A+B}^{mn}, and we are done. If, in another wrinkle, B is found to be contaminated with even smaller particles of another species C, a similar procedure can be applied. One takes the effluent obtained in interval $D_B^{\mathrm{mx}} - D_A^{\mathrm{mn}}$ in the middle segment of E2 and refilters it on the interval $D_B^{\mathrm{mn}} - D_B^{\mathrm{mx}}$ again, producing a Cantor triad. The fractal scheme also provides solutions for cases of overlapping distributions in E2 and E3, each of which creates a Cantor triad.

From the foregoing, it should be clear that a statement or hypothesis is *fractally true* for each iterative step of a series of experiments on which it is consistent with results. It becomes fractally *false* on the first step on which it no longer matches the results. Thus, fractal truth is a provisional, pragmatic, empirical truth. The Hausdorff dimension of the Cantor dust $\frac{1}{\log_3 2} \doteq 0.6309$ may index something like the capacity of the system to spawn alternatives. Higher-dimensional fractals might model systems with additional endpoints. Similar examples could be worked out regarding procedures for creating a work of art, in which there are repeated and refined attempts to achieve or discover an effect.

2.3.5 Similarities to other truth conceptions

Fractal truth shares features with Tarski's [31] theory. The first is that truth is a property not of things but of statements, and statements are "truth bearers." For example, one does not say, ""That cloud is true," or ""That rock is false." Rather, one judges whether the statements "That cloud is gray," or "That rock is hard" are true or false. Statements that are superficially true but become less true upon closer inspection are ubiquitous in science, art, and everyday life. The cloud is gray, and then you spot a few black and white blotches; the rock is hard, and then a few pieces flake off. Fractal truth adopts statements (S0–S2 in Figure 3) as truth bearers. It models the gradual fading of truth and the need for fresh statements upon examination through testing on progressively finer levels of fractal geometry. Fractal truth also adds to the list of possible truth bearers. Non-verbal actions that enact statements or hypotheses, putting them to the test, are included. Examples would be injecting a solution into a patient or irradiating a sample with X-rays in science and hewing marble off a block or dragging a brush across a canvas in art. This accommodates the superabundance of non-verbal expressions in art (and their high frequency in science). The fractal model is a basic attempt to formulate a theory that addresses the complex realities of the laboratory and the atelier.

A second feature that fractal truth shares with Tarski is the use of "truth makers" to evaluate the truth or falsity of statements. In Tarski, truth makers are *metalanguages*. Statements are expressed in a formal language, and their truth is evaluated in a metalanguage. An example takes German as the language and English as the metalanguage. The German sentence, "Schnee ist weiβ" is evaluated in English by "'Schnee ist weiβ' is true iff snow is white". As shown in Figure 3, hypotheses and statements, e.g., H0 and S0, are cast on one level of the fractal framework and evaluated by an experiment, e.g., E1, the next level down. Hence, experiments are the truth makers. Possible objection: Is the meta level not positioned above the statement rather than below it? Answer: Although the scale is smaller on, say E3 vs. E2, E3 can still be regarded as meta to E2 as it is at a higher resolution and was reached over the path of E2 (and all its predecessors). Possible second objection: Is an experiment a "language"? Answer: An experiment can be regarded as a miniature language. It contains "nouns"—e.g., individual pieces of gear, samples, and investigators—and "verbs"—e.g., an instrument stimulates, the sample reacts, another instrument transduces, and a third records. Nouns and verbs are combined into sentences—e.g., investigator A injects the patient with a syringe as the manometer records blood pressure, according to "syntax"; for example, human investigators and automata have agency, while passive devices do not. Cognitive EEG experiments, for example, have been described as communicating with the subject in a non-verbal language [34]. Hence, an experiment in the fractal model might be something like a metalanguage in Tarski.

A third feature shared with Tarski is the concept of *satisfaction*. The criterion of truth for a statement in Tarski is whether or not it satisfies a corresponding expression in the metalanguage in a formal mathematical sense. In the fractal model, a statement or act is true if it satisfies empirical criteria—e.g., produces a certain measurement value or aesthetic effect. This is the pragmatism of the fractal model.

The fractal heuristic may also accord with the familiar concepts of Popper's falsifiability [35] and Gödel's incompleteness [36]. Popper maintained that you cannot formulate a theory from past data that invariably predicts the future, but you can prove theories *false*. He saw science as approximating reality ever more closely by successively discarding falsified theories. A theory that cannot be falsified is of limited value. Popper's theory is like the *via negativa* from Zen Buddhism or Christian mysticism. Falsifiability and *via negativa* are consistent with the metaphorical fractal-like structure of truth and falsity proposed herein. The fractal model entails progressive refinement whereby falsity is an essential component in intimate contact with truth at every level of investigation. A fractal-like epistemic architecture may also be consistent with Gödel's incompleteness theorem. Gödel demonstrated that any logical system powerful enough to be useful is capable of generating statements that are true but not provable within the system. Thus, artificially constructed logical networks and the fine structure of objective truth intertwine, rather like fractal interdigitation of fill and void.

2.4 Testing the fractal hypotheses with formal comparative analysis

We view the fractal entanglement of art and science with truth and falsity as the source of the dictum, "Art is the lie that tells the truth/science is the truth that lies." If this fractal picture is at all apt, then art and science might share properties one can reasonably associate with fractal structures; they should realize these properties in something of an oppositional fashion. This led us to the two hypotheses. H1: Both art and science exhibit one or more general properties of fractals. H2: Art and science manifest these fractal properties in a contrasting foreground vs. background manner. We tested these hypotheses through a formal comparative analysis. Findings appear in Results. Therefore, we identify five general properties of fractals. We illustrate each with one or more familiar fractals. We discern whether each property manifests in art and science and whether the property exhibits a contrasting foreground/background relationship in art *vs.* science. The five fractal properties are the role of dimensions, tendency toward infinity and the infinitesimal, novelty and familiarity, use of ellipsis (exclusion in expression), and the personal. Illustrative fractal examples include the Koch snowflake, the Sierpiński triangle, Pascal's triangle modulo 5, the Mandelbrot set, the Weierstraß function, and the coast of Britain.

Our use of the term "truth" follows a broadly pluralist and contextual perspective in which different domains—scientific, artistic, experiential—operate with distinct yet internally coherent criteria of truth. This stance aligns with established approaches in epistemology that differentiate correspondence- and coherence-based truth in science from interpretive or constructivist truth in the arts. In this framework, our notion of fractal truth functions as a comparative heuristic that highlights recurring structural relations across these diverse truth practices, without proposing a universal or metaphysical theory of truth.

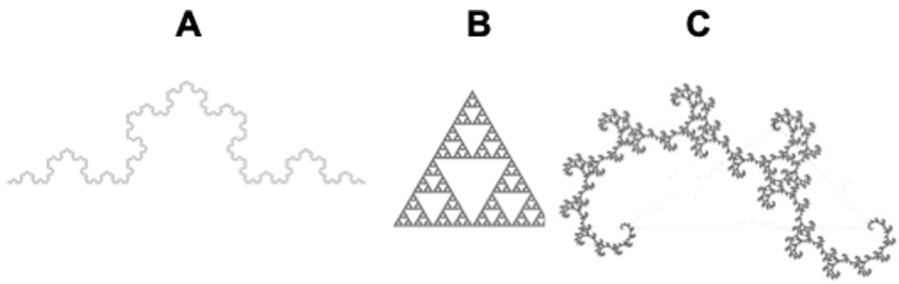

FIGURE 4
The dimensionality of a fractal impacts its structure. Some familiar examples: fractals famously can have *fractional* dimensions, as opposed to the integral dimensions of Euclidean shapes. Small differences in dimensions are associated with radically different forms. **(A)** The Koch snowflake has a dimension $d = 1.2619$ (content provider, Fibonacci; used under the Creative Commons Attribution-ShareAlike 4.0 license). **(B)** The Sierpiński triangle has a dimension $d = 1.5850$. **(C)** The Golden Dragon fractal has a dimension $d = 1.61803 = \phi$, the Golden Mean, suitable for the Golden Jubilee of Fractals celebrated in this special issue (content provider, Prokofiev; used under the Creative Commons Attribution-Share Alike 3.0 Unported license).

3 Results

3.1 Dimensions in fractals, art, and science

What is a *dimension*? Figure 2 illustrates our concept [25]. A dimension, degree-of-freedom, attribute, or quality is a coordinate axis. The axis need not be visible; more often, it is implicit. A dimension has three main elements: somewhere to start (a "0"), somewhere to go (an "∞" or "maximum"), and a size of steps to take getting there (a "1"), as indicated on the axis for attribute x in Figure 2. The direction of the axis conveys a sense or meaning. One or more axes form a *context*, a sense, or a meaning field.

Dimensionality is at the core of fractals. Mandelbrot once even defined a fractal as a set whose fractal (Hausdorff) dimension d exceeds its topological (usually familiar Euclidean) dimension [1]. He later replaced this definition with "a shape made of parts similar to the whole in some way" [37]. Yet, dimensions remain central. Notably, for most fractals, d is *fractional*, although a few have integral d, e.g., the Penrose Tiling [38] has $d = 2$. Figure 4 shows three familiar fractals and their dimensions. These are the Koch snowflake ($d = 1.2619$ [39]), the Sierpiński triangle ($d = 1.5850$ [40]), and the Golden dragon ($d = 1.61803 = \phi$; a variant of the Heighway dragon, [41, 42]). The dimension strongly affects the character of the fractal. Note how radically different the fractals are with small changes in d.

Dimensions represent one area in which to compare and contrast science and art. Art and science are always occupied with which dimensions of a work become conscious and which remain hidden. To conduct a scientific experiment or to create a work of art is to erect a context, a set of dimensions, each with a quantity of a given attribute. Both science and art recurrently measure and create effects along endlessly varying novel and familiar dimensions. As for fractals, dimensionality has a powerful influence on the character of a work of art. Spatial dimensions are a long-standing interest. In theater, stage proportions impact decisions regarding choreography and *mise-en-scène*. More to the point, the early 20th century witnessed an explosion of movements in the visual arts centered on the treatment of 3D space in bold attempts to move away from conventional perspectives. Cubism, for instance, suggested that an object can be viewed from more than one—even infinitely many—points-of-view simultaneously. Among the "isms" of the 20th century—e.g., futurism, making novel interpretations of color, or surrealism, exploring psychological processes—"dimensionism" was specifically dedicated to new dimensions. Its manifesto [43] introduced the N + 1 formula for the arts. This suggests that the arts absorb an extra dimension to accommodate the reinterpretation of space–time in Einsteinian relativity. Such examples make it clear that all artists are constantly varying things along multiple dimensions. Artists even invent novel dimensions, changing what was previously kept constant. Pointillism and futurism in painting stipulated that each color is a dimension. Then, how much *white* is in each color, and how do juxtapositions of color dots and patches change the perception of the color scheme? A novel color-related dimension was introduced. In painting, as in all art, there is endless play with dimensions. As art strives for the unique, it may assemble a previously unknown dimension, call attention to one taken for granted, change the scale of dimensions from the accustomed, and make other variations. This game is never-ending and inexhaustible. Dimensions often occupy the foreground in art.

In science, as in fractals, dimensions are unquestionably critical in both theory and experiment. Every measurement is along some dimension, perhaps a fundamental indefinable of physics (length, time, mass, force, or electric charge); more often, a derived dimension (velocity = length/time, work = force • length, or voltage = work/charge). Fundamental dimensions, by the way, cannot be defined but *can* be measured. Just as art expresses the unfathomable, science measures the indefinable. Science, like art, works and plays incessantly with dimensions. The distinction is that science standardizes dimensions for reproducibility. Derived quantities are defined with exact formulas; units of measure are defined with utmost precision. The constructs and units are supposed to enable reliable measurement, something taken for granted. Although they are useful for solving problems and gaining insights, dimensions are, thus, usually in the background in science. Hence, both H1 and H2 appear valid: dimensions abound in art and science and often assume contrasting foreground/background roles.

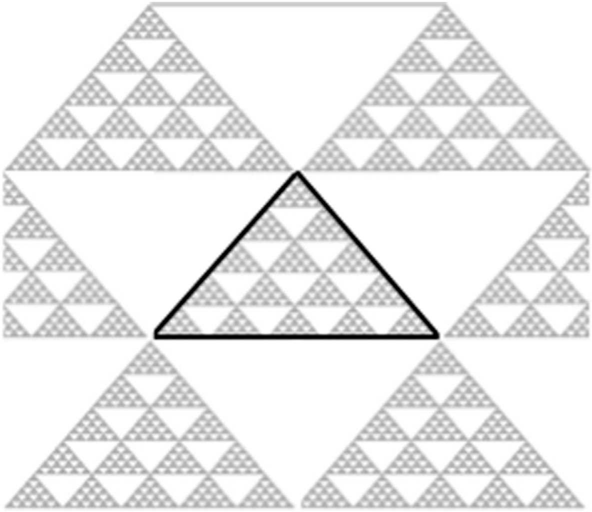

FIGURE 5
Fractals to the infinite and infinitesimal. One level of the fractal Pascal triangle modulo 5 appears in the *bold outline*. The alternating dark and light triangles are reproduced within the outline at smaller and smaller scales, down to the infinitesimal. The fractal is everywhere sparse. However, outside the outline, the fractal may also belong to a network, expanding to infinity. This is a typical property of fractals; they can reach down to the infinitesimal and out to the infinite. (content provider, Alexis Monnerot-Dumaine; used with modification under the Creative Commons Attribution-Share Alike 3.0 Unported license).

3.2 Infinity and the infinitesimal in fractals, art, and science

Fractals concern both the infinite and the infinitesimal. As mentioned, a fascinating, paradoxical property of fractals is that they are everywhere sparse. As shown in Figure 5 for Pascal's triangle modulo 5 [44], a fractal with $d = 1.6826$, when we observe a fractal at an arbitrarily selected level, its generator pattern repeats at smaller and smaller scale down into the infinitesimal. In this manner, the fractal fills out its interior space, yet always leaves immediately adjacent void spaces. One can, however, also think of the selected level of the fractal as being *embedded* in a larger network that repeats the pattern at ever larger scales, moving outward toward the infinite. Fractals span both the infinite and the infinitesimal.

To grasp the tendency of art toward the infinite, consider the *circus*, an art form like theater, only with less scripting and more immediacy. One of the best-known circuses, Cirque du Soleil, has been performing amazing shows for years. However, what is Cirque du Soleil *about*? The huge, busy spectacle—what's the point? The answer: it is not about athleticism although the dancing and acrobatics are highly athletic. It is not about technology although the sound and light shows are technically sophisticated. It is not about entertainment although exciting; some acts are more disturbing and thought-provoking. It is not about money although there is a well-developed business model. No to all these. Cirque du Soleil is about *art*. Moreover, it is about art because art tends to the infinite. If you are about business, profit is the goal. All else can be sacrificed. If you are about athletics, winning is the goal. You may win the 100-yard dash by <1 s, but then how much did you beat the competitors really? If you are about technology, the goal is precision. However, precision soon surpasses what the audience can distinguish. These all manifest the character of science, which tends to the *infinitesimal*. Art, in contrast, tends to the *infinite*. It adds creativity to any enterprise [45, 46]. If you do not make your business about art, it may end-up in one *cul-de-sac* or another. However, art paints its way out of every dead end because it is infinite, like an ever-expanding fractal network.

Art is infinite because it is driven by creativity, and creativity knows no end. It engages infinite aspects of the mind and the world. You have creativity whenever novel or rare (at least to you) solutions are offered. In addition, there will always be novelty. Natural and computer languages are infinite in the number of novel sentences they can generate [47]. In everyday conversation, people readily formulate sentences never before spoken in history. The number of distinct algorithms that can be written is boundless—and so it is with art. For example, people have been devising Internet handles at least since the early 1990s. Personalized license plates are even older. However, we never run out of them—many are still witty, provocative, insightful, or otherwise remarkable. The same holds for bumper stickers, ads, and many other media, even for high art. For art tends toward the infinite, like the new vistas unveiled on progressive levels of a fractal structure. That is why it is wiser for Cirque du Soleil and troupes of every stripe to be all about the art. The infinite tendency of art, moreover, gives you *freedom*. With art, you can entertain—but also insult—your audience. You can make people laugh or cry. You can *scare* your audience, *anger them*, or *disgust* them. You can be morally didactic—or offend morality. You can even *bore* your audience, but not too often. In practice, there are constraints, but in principle, you are not *bound* to anything. Again, options are infinite. You create a product, release it, and let people respond as they see fit. Properly understood, it is liberating to know that, at core, you can do whatever you want and still be an artist—in fact, be living the ideal. Art is freedom and this is very much in the foreground.

Tiny details, the infinitesimal, do capture the artist's attention—but less for their own sake and more for their influence upon a work macroscopically, as distractors from or miniscule contributors to a cumulative effect. For example, numerous tiny imperfections are polished or sanded out of metal or wood for a smooth, shiny overall surface. The infinitesimal is usually in the background in art.

Science, in contrast, tends to the infinitesimal, captured in the German expression "Alle Wissenschaft entartet in Akribie"—"All science degenerates into grunt work." While science would like to seek general principles, in practice it is constantly frustrated. Findings are typically highly specific and circumscribed, often of low relevance. The drug you test does not antagonize all 5-HT1B receptors throughout the body with high selectivity. It *moderately* antagonizes the receptor on some stellate cells—only in the liver, in one line of rats, in some individuals, under pseudo-physiological conditions; i.e., the results carry many limitations and qualifications. For interpretable results, one must painstakingly account for countless factors in design, measurement, data processing, analysis, and other aspects. Since Nature balks at generalization, science tends toward the infinitesimal, like ever-finer fractal levels.

The infinite, however, does inhabit the background in science. While the interpretability of findings is constantly constrained by micro-factors from below, science is repeatedly confronted with infinity from above. Astronomy, for example, once thought the Solar System was all there was to the universe. Then we learned the Milky Way is a galaxy with 100 billion stars, each potentially carrying its own solar system. Then we learned that there are 100–200 billion galaxies! In biology, the archaea were discovered as extremophiles in hot springs and salt lakes, a mere oddity. Archaea have since been found *everywhere*. They are not only the most primordial but also the most numerous and variegated life forms. Our photosynthesis-based lifeform is marginal. Nature continually challenges science to contend with ever-widening bounds, prompting us to revise and refine our conceptions. [48] argued that "the uses of a screwdriver are uncountably infinite" as they are adaptations to unforeseeable future contexts. He deduced that adaptations in biological evolution have the same character, and thus, vast realms of Nature are fundamentally unpredictable. The future is radically open. The same applies to any of an artist's (or scientist's) tools. Hence, creativity is infinite. [49], moreover, distinguished *knowledge* from *wisdom*. The former concerns the past, the facts of what happened; the latter concerns probabilities of what *could* happen. An open future implies there will *never* be an end to wisdom. Hence, purely prescriptive approaches to art and science cannot ultimately succeed. However, in practice, the infinitesimal remains in the foreground in science and the infinite in the background and *vice-versa* for art. H1 and H2 are confirmed.

3.3 Novelty and familiarity in fractals, art, and science

To illustrate novelty and familiarity in fractals, we choose a highly popular specimen of fractal art and supreme example of mathematical beauty, the Mandelbrot set ($d = 2$ [50]). In contrast to many fractals, the Mandelbrot set is drawn on the complex, rather than the real, plane. It is based on the very simple function $f_{\boldsymbol{c}}(\boldsymbol{z}) = \boldsymbol{z}^2 + \boldsymbol{c}$ of the complex variable $\boldsymbol{z}$, where $\boldsymbol{c} \in \mathbb{C}$. Next, we choose the *seed* $\boldsymbol{z} = 0$ and *iterate* $f_{\boldsymbol{c}}(\boldsymbol{z})$. In other words, we calculate $f_{\boldsymbol{c}}(0) = \boldsymbol{c}$, $f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(0)) = \boldsymbol{c}^2 + \boldsymbol{c}$, and $f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(\mathbf{0}))) = \boldsymbol{c}^4 + 2\boldsymbol{c}^2 + \boldsymbol{c}^2 + \boldsymbol{c}, \cdots$. Then, (the hard step) we keep only those values of $\boldsymbol{c}$ for which the infinite series $\left| f_{\boldsymbol{c}}(\mathbf{0}), f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(\mathbf{0})), f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(f_{\boldsymbol{c}}(\mathbf{0}))), \cdots \right| < \infty$. This indicates that every iteration, all the way out to infinitely many, must have magnitude below infinity—i.e., the series is "bounded." Finally, for each $\boldsymbol{c}$ that satisfies that condition, you plot Re{$\boldsymbol{c}$} on the abscissa and Im{$\boldsymbol{c}$} on the ordinate to obtain the fractal boundary (Figure 6). We hope to have clarified the workings of the Mandelbrot set.

The interesting thing is that this simple function and fairly simple procedure yield a curve of extraordinary depth and beauty (Figure 6). Starting with the outermost level, the generator of the fractal, zooming-in around any point of the curve, e.g., $\boldsymbol{c} = (-0.75, 0.1)$, unveils ever-increasing, ever more intricate levels of structure. Successively novel, astoundingly beautiful forms manifest as we penetrate downward. Yet, upon close examination, we always recover the generator on each level. The latter property of fractals is called *scale-invariance*. Furthermore, it is often said that, looking at a fractal, you cannot tell which level you are on. While that is true for many fractals, more generally, one might say each level, viewed in isolation, can look different from its predecessor and successor, but all levels contain the generator somewhere. Above, we observed this applied less dramatically in the case of the Cantor dust. Thus, fractals, in a further paradox, manifest novelty within familiarity. Novelty and familiarity are likewise core aspects of art and science, with each placing a different emphasis.

In art, novelty is in the foreground, and familiarity is in the background. As much as art demands originality and novelty, it rests on age-old themes, references the past, pays homage to prior works, and otherwise venerates what came before. Art does not strive for novel themes—it flourishes by reexamining ideas long interwoven into the human condition: passion and conviction, selfless deeds, undying love, and other dramatic motifs. Rather, what is celebrated is the originality of the *approach* by an individual artist. The revolution is in the unique way of retelling stories as ancient as humanity. In science, familiarity is in the foreground, and novelty is in the background. Science builds mostly on the known but requires some novelty. The novelty can be extreme but is usually incremental. A paper in pharmacology, for example, might show a drug has slightly higher selectivity for H1 than H2 histamine receptors. Novelty is so often minor in science that when the NIH added innovation as a criterion for research grants in the 1990s, it was controversial. Scientists did not want to have to prove innovation. NIH further pushes innovation against resistance by not funding *replication studies*, even though these lie at the heart of science. Science celebrates *novel* discoveries, yet demands replication. Science honors never-before-seen phenomena and demands they be reaffirmed, whereas art repeatedly re-interprets similar themes and prizes novelty in those interpretations. Hence, both art and science call for novelty within familiarity. Overall, hypotheses H1 and H2 hold for novelty/familiarity.

3.4 Ellipsis (the unsaid) in fractals, art, and science

We chose "ellipsis" to describe one property of fractals, art, and science. We use the term metaphorically to mean necessary omission or incompleteness in representation. In art and science, you do not understand what is being said until you know what is unsaid. Ellipsis means that a part is left out of the representation of a phenomenon by necessity, intent, negligence, or otherwise. This is again quintessentially fractal; the voids are as important as the fill.

In fractals, if we recall the Mandelbrot set, how marvelous that its enormous complexity (only a sliver of which is exposed in Figure 6) is all contained in the elementary function $f_{\boldsymbol{c}}(\boldsymbol{z}) = \boldsymbol{z}^2 + \boldsymbol{c}$ (and the iterative procedures). This is the extraordinary *compression* of functional notation. An inch-long string on a page empowers us, in principle, to calculate every feature down to ultra-fine levels. In practice, however, when we draw the set or list its coordinates, we must *stop* at some point. As said for the Cantor dust above, we make an arbitrary cut-off. We refer to this omission of features below the cut-off as ellipsis. It might alternatively be called "approximation" or "truncation" though we avoid these terms as—at least in art and science—sometimes parts are taken out of the middle rather than the tail-end, and sometimes, the loss consists not of mere precision but rather of elements key to understanding. Ellipsis can be viewed as parallel to the fractal property of *lacunarity*. In fractal geometry, lacunarity quantifies the distribution of gaps or voids within self-similar structures [1, 51, 52]. Ellipsis is a structured omission in

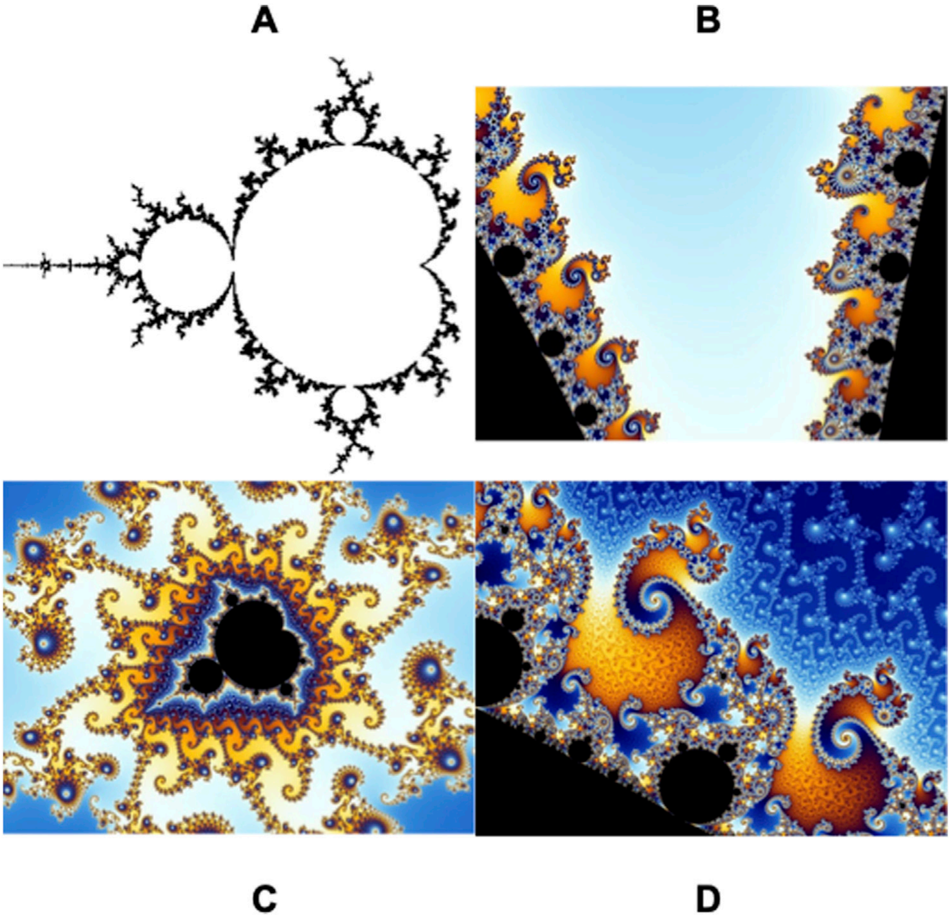

FIGURE 6
Novelty and familiarity in fractals. Progressive magnification of the fractal Mandelbrot set ($d = 2$ [42]) on the complex plane. **(A)** Step 0, generator (content provider, Prokofiev). **(B)** Step 2, descent into the Seahorse Valley about $\boldsymbol{c} = (-0.75, 0.1)$ (colored for effect) (content provider, Wolfgang Beyer). **(C)** Step 7, a jeweled island (content provider, Wolfgang Beyer.) **(D)** Step 11, a row of seahorses (content provider, Wolfgang Beyer.) Novelty of form emerges at each level; yet on close inspection, the familiar generator can always be retrieved. Thus, fractals embody novelty within familiarity (all images were used under the Creative Commons Attribution-Share Alike 3.0 Unported license).

meaning, while lacunarity is a structured absence in space. The concept is further related to the above-mentioned incompleteness of Gödel [36], to tacit knowledge in the sense of Polanyi [53], and to truncation of Shannon information [54].

Ellipsis is nicely observed when plotting the fractal Weierstraß function ($d = 1.5$, [55]; Figure 7). This function is defined as the Fourier series $f(x) = \sum_{n=0}^{\infty} a^n \cos b^n \pi x$ with $0 < a < 1$, b a positive integer, and $ab > 1 + \frac{3}{2}\pi$. Figure 7 shows that, under magnification, in the vicinity of any point there exists a miniature copy of the entire function. The Weierstraß function is self-similar. Thus, when we plot it, even at middle levels, we must *elide* or leave-out essential portions. The Weierstraß function is a classic example of a pathological function that has opened new vistas of fractal understanding.

We now offer examples of ellipsis in art and science. Notoriously, the *meaning* of a work of art is often obscure, left out. Contrary to popular opinion, many artists often discuss, at length, the process and goals of their artistic endeavors. However, the downsides of doing this leave others to elide or avoid explicit discussion of the interpretation of their work. For one thing, audiences may become preoccupied with the explanations *instead* of the work itself. Think of museum visitors summarily scanning the tags, barely glancing at the paintings before moving on! They want to know *who* created the work (the person, discussed below) and *what* is depicted. Furthermore, art benefits from the opportunity that anyone can assign a private meaning to a piece. This process goes better if it starts with attending to a work of art *first* and mining its hidden meaning *later*. Artists want you to invest effort into a work to grasp its meaning. Like anything in life, you prize the work more if you put something into it. Hence, ellipsis is in the foreground in art.

Science, in contrast, is *supposed* to say what it means completely and unambiguously. Yet, scientists, too, often leave key parts unspoken. This happens especially in pedagogy, in textbooks and lectures, but also in research papers. Examples include the teaching of redox reactions in chemistry. As a student, one learns that when a molecule gains an electron in a chemical reaction, that is "reduction." However, the origin of the term is rarely explained. It is a reduction because electrons bear a *negative* charge, so adding one *decreases* the total charge on the target. When they advance to organic chemistry, students learn that the criterion for reduction suddenly becomes the gain of a *hydrogen atom* (H). The reason is rarely explained. It is

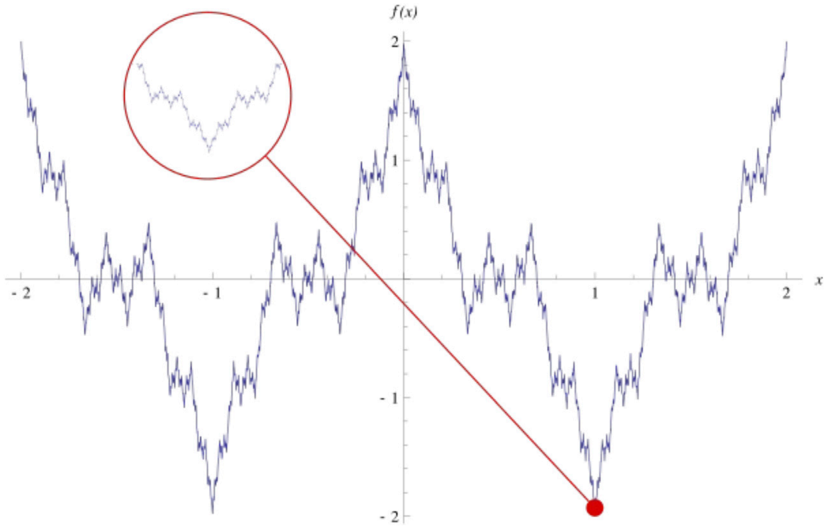

FIGURE 7
Ellipsis (arbitrary cut-off) in fractals (the Weierstraß function). The fractal function ($d = 1.5$ [55]) is the Fourier series $f(x) = \sum_{n=0}^{\infty} a^n \cos b^n \pi x$ ($0 < a < 1$, b is a positive integer, $ab > 1 + \frac{3}{2}\pi$). The blow-up of the *red dot* at the local minimum shows that the vicinity of each point on f is similar to the entire function. In plotting f, we exercise ellipsis, and we leave parts out.

that the H gained carries with it an electron, so it is still a reduction in the original sense. However, the extra rule that, when adding an electron, it is okay if *something else*, like the proton in H, comes along for the ride, is elided. These are key omissions.

Other pedagogical examples come from classical electrodynamics. Physicists are fond of discussing *electric dipoles*, pairs of charges of opposite sign $+q, -q$, separated by distance d. However, they rarely explain why the mutually attractive $+q$ and $-q$ do not smash into each other, ending the dipole. The answer is that there is usually *something between* the two charges that *holds them apart*. That could be, for example, the rest of a molecule, as in hydrogen chloride (HCl), the phospholipid membrane for intracellular–extracellular dipoles, or innumerable other structures. In other words, the dipole is typically treated as a problem of free-body electrostatics but is actually a problem of electrostatics with (implicit) boundary conditions. A factor essential for complete understanding is omitted. In a second electrodynamic example of ellipsis, textbooks typically emphasize that all free electric charges inside a spherical conductor always redistribute themselves uniformly on the surface. However, they rarely explain why the charges (assumed positive and mutually repellent) tolerate sitting next to each other on the surface. The answer is the unstated assumption that the sphere is embedded in a *non-conducting medium*, e.g., room air. Although the charges dislike being next to each other on the surface, they dislike jumping off into a non-conductor even more. Gathering on the surface, they are as far apart from each other as they can be *without leaving* the conductor. An essential boundary condition again goes unstated in this common ellipsis. We understand that ellipses exist in science, but it takes much contemplation to identify them. Hence, they are in the background. H1 and H2 are again verified in that ellipsis occurs in both art and science, more often in the foreground in one and in the background in the other.

3.5 The personal in fractals, art, and science

We speak of the personal in fractals, art, and science in two ways. The first is in the everyday sense of the word. The character and station of individual artists and scientists influence their oeuvre and its reception. Although a formal term, the personal in the second sense is not a measurable or physical variable. Rather, it is an interpretive or epistemic dimension. It refers to the creator's or investigator's and the observer's role in artistic and scientific production and appraisal. Since artists, scientists, and their public all act as individual human beings, in scope the concept invests design, creation, and impressions of artworks, and, respectively, design, conduct, and impressions of experiments (Steps 1,2 and 6 of Table 1).

In fractals, as in any branch of austere mathematics, the everyday personal is far in the background. Mandelbrot did, nevertheless, conclude his masterpiece [1] with biographical sketches of people behind the mathematics—that is, a touch of the personal. [56]'s recent review included photos or portraits of fractal founders, and hence, the veneration continues. In the formal sense, the above phenomenon of ellipsis indicates that we usually leave something *out* of fractal representations. That implies a personal choice by the analyst. Such choices have practical consequences. An illustrative example is the famous fractal question, "How long is the coast of Britain?" [57]. As is often found for fractal geometries, the answer depends on the size of your *ruler*. A 20-km ruler delivers one value. A 10-km ruler delivers a larger value. A boulder-sized ruler delivers a much, much larger value. A ruler the size of a grain of sand yields an enormously larger value. The analyst makes the choice of the measurement scale. We pointed out [58] that this is similar to measurement in quantum mechanics, where the investigator's arbitrary choice of apparatus under Process 2 impacts the eigenvalues measured in a quantum experiment. Hence, the personal in the formal sense is significant in fractals and science, though rather in the background.

A quote attributed to Madame Curie «…qu'en matière de science l'on doit s'intéresser aux choses et non aux personnes» [59] translates to "in science, we are interested in things, not people." The personal in the everyday sense is not *supposed* to matter in science; in art, in contrast, it is *expected* to matter. In art, personality is in the foreground, and in science, it is in the background. To encounter, for example, a neon-lit marquee outside a scientific congress advertising a talk by a leading scientist would be crass. Though inside one might well enjoy the good professor's grandstanding, faux modesty, and cutting remarks, that is not supposed to be the point. In art, in contrast, crass self-promotion is commonplace. Art *can* be impersonal, yet often is highly personal to a degree unseen in science. Experts recognize, e.g., brush strokes of individual painters, even as they vary across periods. The artist is expected to play the prima donna. It is widely recognized that the price of an artwork may go more by the artist's name than its quality.

Moreover, as [35] and [27] remarked, science is fundamentally undemocratic. The opinion of a respected authority outweighs that of dozens of routine technicians. In that regard, the personal *is* relevant in science. However, if one scientist has an idea in accord with Nature, it ultimately does not matter if everyone else disagrees. Moreover, with respect to essential content, even top-ranking scientists are expected to keep their personalities out of their work. Science eschews the idiosyncratic, and art embraces it.

We now offer a few more reflections on the personal in its formal sense. Art and science represent different ways of uncovering truth about the world and the human condition. Artists, for instance, aim to create works that, through expressing a certain personal state-of-mind, resonate with their audiences, even to the point where a particular work becomes "my song," "my poem," or similar. The process begins as the artist chooses to render a personal interpretation on a theme or subject. The process of creating the piece translates the artist's inner thoughts and emotions on the subject into the final product. The resulting work evokes emotions and thoughts in the audience. The catalyst for these responses is the highly individual interpretation by the artist. This is what we mean when we say the personal is an interpretive and epistemic dimension. Note that the value of a piece is most often determined by the *style* of the artist. In art, the goal is to express a unique *point-of-view* since art is rarely concerned with discovering new themes or subjects. One may consider such truths to be highly subjective as both artist and audience join in the process by sharing personal thoughts and emotions. This brings us to the objective aspect of art-discovered truths. Human minds are "a case series of one." No one else can access the inner workings of my mind, just as I cannot access anyone else's. Art, therefore, has the universal ability to evoke corresponding thoughts and emotions in otherwise strictly separated minds, thus revealing truths about the human condition. Such truths are largely established by recognizing that particular artistic expressions evoke kindred emotional responses and instill parallel lines of reasoning (e.g., funny stories bring smiles and laughs, dramas make us weep, mysteries let us marvel at the powers of observation and strong logic of the detective, and other reactions for other genres). The larger the audience—with the Internet now reaching millions—the more palpable the truth might be. Science, in turn, establishes aspects of reality that are unaffected by the emotions and opinions of individual humans. For the scientist, the process may start with hypotheses to be confirmed or refuted by experiments. The results are untainted by the scientist's thoughts and emotions. The results are considered facts to be confirmed and replicated and possibly amalgamated into theories, leaving individual preferences aside. Thus, regardless of individual mind states, science-established truths should remain unchanged if correct procedures are carried out. In summary, art reliably shares subjective thoughts and emotions across individual human minds, confirming those minds share commonalities; similarly, science shares objective truths across minds, confirming epistemic communication is possible. Art allows us to share experiences, whereas science allows us to share facts. Even formally, the personal is in the foreground in art and in the background in science. H1 and H2 are again upheld.

4 Discussion

4.1 Summary of findings

This paper investigated the research question: What is the epistemological relationship between art and science? We argued that in art, falsity is in the foreground, and truth is in the background; on the other hand, in science, truth is in the foreground, and artifice is in the background, summarized by the dictum, "Art is the lie that tells the truth/science is the truth that lies." We suggested that truth in art and science might be modeled by a fractal metaphor and worked out an example of elementary formalization of "fractal truth." We cast hypotheses that, if art and science have oppositional fractal character, they should exhibit one or more general properties of fractals, and these fractal properties should manifest in a contrasting foreground/background manner. We tested these two hypotheses by a formal comparative analysis of art and science with respect to five fractal properties: dimensions, the infinite and the infinitesimal, novelty and familiarity, ellipsis (necessary omission), and the personal (as an epistemic and interpretive dimension). We found that all five properties were evident in both art and science. The foreground/background prominence of the property was opposite in art vs. science in each case. These findings suggest a fractal heuristic may be apt for characterizing art and science as disciplines and their mutual relationship.

4.2 Comparison with prior studies

Recent work varies widely on the connection between art and science. One distinguished expert [60] believes that it is impossible to generalize relationships between art and science at all, as neither is fully defined nor homogenous. We respectfully disagree. We find no reason that the properties of art and science identified in this paper would not apply widely across art and science despite their internal heterogeneities. Another expert (Love [12]) is unimpressed by past attempts, somewhat in the present vein, to analogize the working processes in art and science. When he divides the working process into *observation*, *generalization*, *and testing*, he finds that artists rarely generalize, while scientists often do and that testing differs widely between art and science. However, what we regard as generalization and testing is not what Love views. He refers to generalization in theory building. In this regard, we find generalization is also rare in scientists, typically reserved for the terminus of a long program of investigation. Meanwhile, both we and Love find another type of generalization common in artists and scientists. That is the habit of building templates from experience for future pattern-matching. Testing in Love refers to acceptance of a work by the professional community. In our fractal model, testing occurs on the level of single artworks or experiments where we find commonality between art and science. Thus, on one plane, Love finds differences between art and science, and on another, we find similarities.

Other thinkers identify *separate* truths for art vs. science. An older position is the "two cultures" view, after the celebrated lecture [61]. This perspective contrasts the prizing of personal experience and idiosyncratic knowledge in art with the reverence for impersonal objectivity in science. Our model highlights the greater salience of personal experience in art while noting that it is not absent from science. Moreover, we indicate that the artist is as bound as the scientist by material objectivity. [16] argued art and science seek different types of truth, systematically authenticated external truths in the case of science and subjective truths unique to the individual in the case of art. Our fractal model treats truth as the same for art and science; the foregoing difference arises in the impression. The methods of art and science can be applied both introspectively and extrospectively. The above-described ubiquitous resonance of sentiment that art invokes in one mind from another and the power of symbols to transmit meaning socially (detailed in [13]) belie the concept of individual truth.

Finally, several authors find common truth between art and science [11, 14, 62, 63], or even that truth in art supersedes that in science [15]. The famous chaos theorist [62] opined that the technical trappings of science obscure its underlying kinship with art. We concur. [64] stipulated that "art and science are both about observation and interpretation." We agree, although this leaves uncommented the prodigious activity involved in creating artworks and conducting experiments. [65] proposed a codependence thesis: scientific cognition depends on artistic innovation and aesthetic skills, while artistic creativity depends on scientific innovation. We find the concept congenial. We add that every productive human endeavor employs elements of both art and science; the former especially for heretofore unsystematized aspects. Going past codependence, we state here that art and science are, at heart, the same endeavor, only with different emphasis. Foss [14] rejects the belief that science is the measure of all things, relegating art to aesthetics alone (scientific realism). Noting that art functions largely through non-verbal symbolism, Foss instead adopts symbolic realism and finds no substantive difference between art and science in articulating or measuring the world. [63] shared the sentiment of [62] and symbolic realism. Our fractal picture aligns with these thinkers.

Several particular aspects of the present fractal model were invoked or approached in prior work. [13] adopted Frege's definition of a "sign" as "any expression enlisted to designate an object." This is in harmony with the inclusion of non-verbal actions as truth bearers in the fractal model. [14] also viewed this as part of symbolic realism. [63] found overlap in the cognitive skills of artists and scientists, consistent with the similarity of the working process in the fractal model. [16] agreed with the provisional character of scientific truth inherent in the fractal model that it only approximates reality. [66] similarly observed that scientific theories enjoy a limited lifetime. Yet not all the past is discarded as science values stability as a criterion of truth. This accords with our finding of novelty as a *background* trait of science. Similarly, [67] observed that the part played by new discoveries is overstated in science; most interesting results involve established phenomena in new contexts, similar to our fractal view of art. [68] emphasized that art often goes beyond evidence, even against counterevidence. [12] noted that even a scientific theory that has been disproved (famously, Newtonian gravitation) can still be useful. [11] noted that scientists, too, often scorn facts, comparing Gedankenexperimente to poetic fictions. All these points harmonize with the concepts of art within science/science within art, the intertwining of truth and falsity, and the utility of the artificial void frame in this study. Overall, our fractal model resonates with and remains defensible in light of much contemporary art–science scholarship.

We note that the phrase "fractal truth" and related metaphors have appeared previously in popular, spiritual, and interdisciplinary literature ([69, 70]) and that a related notion of "fractal epistemology" has been developed in transpersonal and psychological scholarship [71]. These prior uses are largely rhetorical or situated in the humanities or social sciences rather than in mathematical physics. Our present contribution differs. We frame fractal truth explicitly as a comparative heuristic, offer a simple formal illustration (Cantor dust/shallow-bed filtration), and propose structured hypotheses (H1 and H2) amenable to empirical or formal development in future work.

4.3 Possible quantification of concepts

We offer a rough concept of how our notions of a fractal character of science and art might be quantified and even evaluated empirically. For science, recall the fMRI experiment of Figure 1. We pointed out that fMRI methodology involves quantitatively examining a series of smaller and smaller effects involving the magnetic resonance (MR) signal. MR itself is already a second-order effect. Very strong magnetic fields (e.g., 3 T) must be applied to a subject's brain to force a minimal ($\sim 7/10^6$) fraction of the available H atoms to realign their magnetic fields, yielding a bulk magnetization vector in the brain that is very small but large enough to image. Furthermore, fMRI does not even consider the whole vector, only the influence of $T2^*$ relaxation on its transverse component. BOLD is only part of the $T2^*$ effect. Then, one attempts to remove the hemodynamic response from the BOLD signal to leave the part due to neuronal activity. Then, one seeks (in the case of [22]) to access the sub-portion due to speaking one word by one subject minus background brain activities. Then, one averages the weak signal across many words and then across many subjects. Thus, we obtain tertiary, quaternary, and higher-order effects, representing smaller and smaller pieces of the original signal. This recalls the shorter and shorter line segments in a fractal structure as one zooms in. Thus, one approach to quantifying ideas in this paper might be to examine the progressive diminution of a neuroimaging signal after successive methodological interventions (filtering, averaging, discarding, *etc.*) and observe whether fractal patterns emerge—a very rough conception.

We said that art has an opponent fractal relationship to science, representing the void. For a work of art, we might expect the opposite of what we described above for science. Adding detail to art might *strengthen* the effect achieved. If realism, for example, is desired, adding lighting, textures, shadows, and fine features to a portrait might progressively increase the realism experienced by a viewer. If an emotional response, e.g., melancholy, delight, fear, or otherwise, is sought, this might be achieved by progressively adding judiciously chosen colors, proportions, symbols, and other elements, eliciting larger and larger responses as detail is added. The sizes of the effects might be measured using subjective rating scales, polygraphy, and other means. Again, one could search for fractal structures in the pattern.

4.4 Possible applications

We provide a few speculative ideas on possible applications of fractal truth in physics. Areas where present concepts might be useful include complexity [73, 74], multiscale analysis [75, 76], turbulence [77, 78], and statistical self-similarity [79–81]. Our paper offers only a primitive formalization of fractal truth (Figure 3). More advanced versions would model multiple endpoints with higher-dimensional fractals. As mentioned, fractal parameters in such models express the capacity of a system to spawn alternatives. They may permit estimation of how many truth/falsity evaluations in a series are needed to meet fixed pragmatic ends. Coupled validating statements as in Figure 3 may reveal covert fractal patterns in systems permitting description in more compressed notation. Interestingly, in Figure 3, the discovery of novel (possibly unwanted) system features such as A + B aggregates, extra species C, and overlapping particle size distributions triggered propagation of the fractal pattern downward, as opposed to termination. Such features might be fruitful in the above branches. In complexity theory, for example, [72] proposed an impressive model of truth and certitude; incorporating fractal truth might add refinements. Multiscale analysis concerns systems with multiple time, length, or other scales possibly obeying separate laws. This might be characterized by fractal validation statements that are T on one level and F on another. In the presence of perturbations, turbulent eddies and other structures emerge when energy, momentum, or angular momentum flow into a volume faster than they dissipate, and the excess channels into novel modes. Fractal truth models could map out profiles of possible eddy sizes, and one could calculate their distributions. Or propagation of fractal patterns might signal the presence of unexpected perturbations. Finally, fractal truth analysis with appropriate validation statements might uncover statistical self-similarity—in systems where it was previously unrecognized.

5 Conclusion

According to our formal comparative analysis, art and science both possess multiple fractal properties; these properties are present in a contrasting foreground/background manner. This is consistent with a picture of any work of art or science as a fractally entangled mix of truth and artifice. Art is the lie that tells the truth/science is the truth that lies. Science and art have differences to respect, but they are close enough to teach each other [82, 83]. From the scientist, the artist may learn to be systematic—to ignore the irrelevant in pursuing a common thread. The scientist may learn from the artist to integrate the unpredictable—that creativity is endless, even in Nature.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material; further inquiries can be directed to the corresponding author.

Author contributions

JON: Conceptualization, Writing – original draft, Writing – review and editing. II: Conceptualization, Writing – original draft, Writing – review and editing. GP: Conceptualization, Writing – review and editing. MJ: Conceptualization, Writing – review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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