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Engine sounds reflect a racecar driver's cognition

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We analyze the engine noise of racecars to shed light on the interaction between the brains of the drivers and their racecars and also the interaction between the brains of different drivers for the International Automobile Federation (FIA) Formula 4, E4 Championship. Statistical analysis is performed using the same theoretical tools as those adopted in the recent past to study the brain of an orchestra director through the resulting music. The result of this statistical analysis is the evaluation of a scaling parameter that we compare between drivers. We interpret this scaling parameter as a measure of the driver's ability, with 1 representing maximal adaptability and 0.5 representing random or minimal adaptability (less than 0.5 does not exist for the trajectory model we have). The results obtained show that higher values of the scaling parameter, measured in a single qualifying lap, correspond to better performance in their championship. We also study the training process that allows novice drivers to move from values of the scaling parameter around 0.7 to values very close to 1 as they gain experience. We find that more experienced drivers have a larger scaling parameter and we also explore the effects of competition that can lead to a decrease of the said scaling parameter. This is in line with phenomenology theory, despite being temporary. This work suggests that the study of racecar noise can shed light on the difficult issue of cognition. Having in mind the therapeutic applications of music, we conjecture that this discovery may provide an important contribution to rehabilitation therapy. We also contribute to the emerging field of human-machine interaction by showing how to transmit crucial events to a machine and detect them.

KEYWORDS

scaling parameter, complexity matching effect, ergodicity breaking, cognition, human-machine interaction, racecar driver

1 Introduction

This paper is devoted to the statistical analysis of the noise of racecar engines, and just to be clear, it is our position that this so-called 'engine noise' contains a great deal of information about the driver of the racecar. What we commonly think of as engine noise is actually a reflection of the choices being made by the racecar driver as the limits of the racecar are tested within the confines of a racetrack. It is our contention that the sound of the engine mirrors the actions of the driver's motor control system, which because the driver is in a race the typically unconscious behavior of driving, say on a sunny day along a straight, flat, open highway, is raised to the level of consciousness, as would occur if a cloud burst of April showers coincided with a meandering road into the foothills.

The driver in both scenarios would switch from their usual relaxed unconscious driving habits in the sun to the more sharply tuned responses of their conscious driving habits in the rain. This shift from the unconscious to the conscious functioning of the brain is readily understood using the two system model of the human brain hypothesized by the winner of the 2002 Nobel Prize in Economics, Daniel Kahneman. He describes the two-system brain in his remarkable book, *Thinking, Fast and Slow* [1] and attributes the origin of the terms to K. Stanovich and B. West. In his book Kahneman describes the two brain systems as.

- System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control.
- System 2 allocates attention to the effortful mental activities that demand it …The operations of System 2 are often associated with the subjective experience of agency, choice, and concentration.

Our purpose in introducing the two-brain model here is part of an attempt to contribute to the progress on the open issue of a data-supported theoretical interpretation of cognition. This is a "hard" problem in the sense of Chalmers [2], who coined the term "hard problem of consciousness" to distinguish the totality of consciousness from the easy problems that are amenable to reductive logic. This requires us to provide the reader with a clear illustration of a number of important theoretical concepts for characterizing a stochastic time series *X(t)* generated by a complex phenomenon of interest. The phenomenon of interest here is the sound of a racecar engine during a race.

1.1 Scaling

We begin with the assumption that the racecar and its driver constitute a complex system in the sense defined by N. Wiener in his groundbreaking book that birthed the science of *Cybernetics* [3]. The existence of a quantitative measure of the undefined quantity "complexity" implies that there must be an underlying theory. Anderson maintained that complexity results from the fact that *more is different* [4] such that as a system becomes larger and larger there is more opportunity for behavior to emerge that could not exist in smaller (simpler) systems; see West and Grigolini [5] for a discussion of some of the nuances associated with defining terms. We have found it convenient to define the quantity complexity by a class of phenomena whose empirical time series scales such that, for a constant scale λ the empirical time series obeys an equation of the form

$$X(\lambda t) = \lambda^{\delta} X(t), \tag{1}$$

and δ is a scaling parameter that measures the level of system complexity. This paper is about the implications of this equation and how we can process the empirical data to determine the mannner in which the scaling parameter tracks the level of system complexity.

If Equation 1 is so important how do we unambiguously interpret it? The most common way to interpret the scaling relation is through the phase space probability density function *P(x,t)*. The probability that the random variate $X(t)$ lies in the phase space interval $(x, x + dx)$ at time t is given by $P(x,t)dx$. One way to obtain the PDF from the data in by using the empirical time series $X(t)$ to construct a diffusion process under the assumption that the time series scales to obtain the empirical probability density function:

$$P(x,t) = \frac{1}{t^{\delta}} F\left(\frac{x}{t^{\delta}}\right). \tag{2}$$

where $F(\bullet)$ is an unknown probability density funciton in general. This scaling probability density function is "scale-free" in that the variable $y = x/t^{\delta}$ is dimensionless.

Consider a particle moving with constant velocity, without change of direction; it will explore distances proportional to times, producing the maximum scaling parameter of $\delta = 1$. The adoption of renormalization group theory [6, 7] leads to a more appropriate mathematical definition that yields a scaling parameter of $\delta = 0.5$ for frequent uncorrelated changes of direction. Note that this value of the scaling parameter means that the underlying process is a simple diffusion, and the unknown probability density function becomes a Gaussian distribution. To obtain scaling parameter values, we use the method of diffusion entropy analysis [8–10] as described in Section 2.3.

1.2 Multiscaling and ergodicity breakdown

It is necessary to interpret the time series associated with anomalous diffusion processes, that is, for non-Gaussian processes regardless of the value of their scaling parameter δ. Even in the simple case where with each step the system must choose randomly between opposite directions, the process does not generate only a single scaling but remains scale-free. Thus, multi-scaling or multiscale processes are ubiquitous in complex phenomena and are given by fractals, such processes are typically time dependent, and are also denoted as multifractals, as explained, for example, in Allegrini et al. [11].

In the case where the time interval between two consecutive changes of direction is characterized by a waiting–time probability density function with an inverse power law index $\mu < \infty$, a new important property emerges. Time series with an inverse power law index in the interval $1 < \mu < 2$ is not ergodic, whereas those with the inverse power law index in the interval $2 < \mu < 3$ are ergodic and, as is well known in Classical Statistical Mechanics, an ergodic process has time averages and ensemble averages, which are equivalent. However, the non-ergodic time series are non-stationary, so the two time correlations of the fluctuating velocity $\dot{X}(t)$ are no longer stationary. That is, $< \dot{X}(t)\dot{X}(t') >$ where the brackets denote the averaging process and the two time correlation depends on the two times t and t' separately and not on the time difference given $|t - t'|$. This property is called aging or weak ergodicity breaking. The non-stationary properties become evident when $\mu < 2$ and $\mu = 2$ is the border between the ergodic and non-ergodic time series. From the vast literature on ergodic and non-ergodic processes, we invite the readers to consult [12–15], which are reliable examples of a correct approach to these complexity phenomena.

1.3 Complexity matching effect

The complexity matching effect was introduced by West et al. [16] as one way to understand the ubiquitous aspects of complex networks such as the appearance of non-stationary and non-ergodic statistical processes and inverse power law probability density functions. They reviewed the traditional dynamical and phase-space methods for modeling such networks as their complexity increases and focuses on the limitations of these procedures in explaining complex networks. Of course they were not be able to review the entire field of network science, so they limited themselves to a micro-review of how certain complexity barriers have been surmounted using newly applied theoretical concepts such as aging, renewal, non-ergodic statistics and the fractional calculus. One emphasis of their review is the transport of information between complex networks, which requires a fundamental change in perception that we express as a transition from the familiar idea of stochastic resonance to the newer concept of complexity matching.

The complexity matching effect is an interesting phenomenon generated by aging and weak ergodicity breaking that forces us to make hypotheses (falsifiable conjectures) about the challenging issue of cognition [17–21]. These papers address the important issue of how complex processes characterized by aging and ergodicity breaking respond to perturbations that we may use to aid in developing an understanding of these complex processes. The answer to the question afforded by this research work is that these complex processes respond only to perturbation time series with the same (or higher) complexity. This is the reason why the term "complexity matching effect" was coined. It is important to note that the theory used to explain the complexity matching effect is the popular linear response theory of Kubo et al. [20]. The theoretical foundation of the linear response theory is quantum mechanical. The work on complexity matching, especially that of [19] is based on the conjecture that quantum mechanics may be compatible with the existence of ergodicity breaking; see also the appendix in West and Grigolini [5].

This is where this paper establishes a connection with the open issue of cognition [21, 22]. We hypothesize that the approach taken herein, based on the assumption of ergodicity breaking that is compatible with quantum mechanics, may be a bridge between the arguments adopted by Tononi [21] and those of Faggin [19]. This paper is intended to contribute a new dimension to the discussion on the validity of this conjecture.

Now that we have illustrated the important scientific/mathematical concepts utilized in this paper, we find it necessary to also illustrate the key conjectures that we introduce here. These conjectures are explicitly fleshed out in Section 1.4 and Section 1.5.

1.4 The brain-engine analogy; for better or worse

In this section, we introduce an unlikely analogy between musicians playing musical scores in an orchestra and professional racecar drivers competing in a race on a professional racetrack. We selected music as the field most studied from a science perspective, particularly from the perspective of the music being complex, as we shall briefly review.

Stewart et al. [23] studied neuroplasticity in the brains of musicians. They also examined the motor and sensory abilities of the musicians. This is not unlike the motor and sensory abilities required to drive a professional racecar [24]. Additionally, the human brain has recently been compared to the role of an orchestra conductor [25] conducting music, interpreted as mirroring the mind [26] of its composer and player. Pease et al. [27] found complexity within human performances, noting a difference in the complexity measures between computer-played and human-played performances of the same musical score. They determined that the complexity for a computer-conducted piece is significantly lower than that for a human-conducted performance. In the case of human-conducted performance, we achieve more than one scaling parameter [11]. These multiple scaling parameters are defined as multifractal dimensionality [28–33], which we use here as a working measure of complexity.

In the present study, we assume that a racecar engine in idle mode is analogous to a computer-played piece of music, while an engine responding to a racecar driver exhibits a level of complexity analogous to a human-played piece of music that is above and beyond the computer-played same piece of music [26, 34–37]. We further expand on this analogy by analyzing the complexity of the sound of the idle engine as well as the driven engine, using diffusion entropy analysis, and obtain the scaling parameter "δ" [8–10]. From our perspective, the scaling parameter reflects the fact that while the computer simply plays the notes as written, without interpretation, humans bring their knowledge, experience, and feelings to the performance [26]. These factors play a role in the analysis of "changes" in their play versus "changes" of a computer performing the same score. It is important to note that here we analyze the "changes" in the frequency of the engine pitch (engine music) as a direct response to driver's behavior.

A driver in this study is viewed as analogous to a musician playing an instrument called a "racecar." A racecar driver performs on a track in the same way that a musician performs a piece of music. Shifting gears at specific points on the circuit is analogous to striking the piano keys, plucking guitar strings, or bowing a violin, for each instrument being guided by the musical score. In this way, using the "musical instrument" that is, the racecar, the driver performs a "composition."

However, we cannot ignore the not-so-subtle difference between professionally driving a racecar and professionally playing an instrument. The common characteristics of a high-level athlete (that is, a professional racecar driver) and a professional musician are that both are highly specialized activities that require extensive training, so the literature describes associated specific neural substrate modifications, both in the context of motorsports [38, 39] and music performance [23, 40]. The brain adapts to the unique and demanding requirements of these distinct complex tasks and becomes specialized in managing a definite tool such as a piano, a guitar, or a racecar. Mastering these instruments develops unique skills and their sounds reflect information about the player or driver, as they directly result from individual, intentional actions, such as shifting gears and breaking into curves.

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The brain appears to have the ability to specialize in the management of a definite tool such as a piano, a guitar, a drum, or a racecar. Beyond the particular skills developed to master one of these instruments, they produce a sound that necessarily carries information about the player (or the driver) since it is the direct effect of the individual and "voluntary" acts of a single human interacting with the tool (or the machine). Specifically, this study focuses on observing and analyzing these individual acts in order to study possible differences among players (drivers) while proposing an interpretive hypothesis. It could be argued that the performance of a musician and a racing driver differs fundamentally, e.g., in terms of physical demands, where driving a Formula 4 car subjects the athlete to significant physical strain due to the vehicle moving, which involves strong accelerations as well as various mechanical stresses [34, 35].

Auto racing poses unique environmental stressors (e.g., heat, humidity, exposure to toxicants, noise, g-forces) and cognitive (competition stress, focused attention, fatigue) and physiological (cardiovascular, muscular) challenges for racecar drivers [41, 42]. Steering on road courses (Formula 4) is on average 157 N per turn, while the brake pedal forces range from 600 to 1,200 N [43, 44]. Acquiring the skills and expertise of racecar driving and developing the adaptive cognitive-motor control mechanisms, endurance, and strength required by racing require high levels of cognitive task organization and complexity of the hierarchical organization of neurophysiological and motor control systems. The driver of an average passenger car was discussed in detail in the work of Lohani et al. [45]. Two other research groups [46, 47] contributed to the analysis of the driving workload. They also discuss the workload of an ordinary driver.

The average passenger car driver does not experience the same difficulties and precision movements that a racecar driver experiences, as emphasized in Section 2.1, which describes in detail the demographic and skill required by a racecar driver. This emphasizes the skill necessary to drive a racecar, which might include stress [48], similarly to the skill necessary to play a musical instrument at the highest level. The authors believe that workload research might have an important connection to dealing with stress and control factors [49].

1.5 Quality of the brain-engine analogy

Similarly to the conjecture made by Vanni and Grigolini [26] and Pease et al. [27] and verified that human-played music is more complex than a computer-playing the same music. Here, we conjecture that a driven racecar exhibits a higher degree of complexity than does an idle racecar. We conjecture that, as in the case of music, the source of the added complexity is the activation of cognition in the racecar driver to control the instrument.

In the present study, the racecar driver is viewed as a musician playing an instrument called a "racecar." A racecar driver performs on a track in the same way that a musician performs a piece of music. Shifting gears at specific points on the track is analogous to hitting the piano keys, plucking guitar strings, or bowing a violin, each movement being guided by the musical score. In this way, using the "musical instrument" of the racecar, the racecar driver performs a "composition" In this work, the empirical scaling parameter δ is interpreted as a unique measure of the complexity of each member of a group of drivers from the Formula 4 E4 Championship of the International Automobile Federation (FIA).

It is important to emphasize that the racecars used in this study feature a manual transmission that requires conscious decision-making (System 2 in the two-brain model), in contrast to an automatic transmission which is electronically controlled and operates independently of the driver. In the musical context, a computer plays a musical score with less complexity than an 'intelligent' human musician [27]. In the context of racecar drivers, the scaling parameter δ is determined by different levels of performance, and in this sense, driving a racecar would activate cognition in the same way as playing a musical instrument.

1.6 Organization of the paper

This article is organized as follows. In Section 2 of Statistical Analysis, we explain how we obtain our data and how we analyze them using DEA. Cognition generates an ergodicity breakdown (as described in Section 1.2). Following the earlier work of Pease et al. [27] in Section 2.2, in which we interpret this effect as the occurrence of crucial events. It is important to note that according to Allegrini et al. [10], crucial events are invisible, much as a person becomes invisible on Time Square in New York on New Year's Eve. The problem of detecting unseen crucial events was solved through the development of DEA, as we subsequently show in Section 2.3. In this section on finding and analyzing crucial events, we indicate the conditions under which the observational data were collected and explain why crucial events become visible based on the new analysis of the properties of Lévy walks to analyze the engine pitch (engine music) of racecars using the DEA method to detect otherwise invisible crucial events. This makes it possible for us to activate this method as a proper way to deal with the racecar problem and calculate a scaling parameter δ for each driver, which provides a quantitative measure of the degree of complexity associated with the particular driver-car symbiosis. The results of the championship races are discussed in Section 2.4.

In Section 3, we analyze the training process and how it applies to the "learning curve" of the driver. In Section 4, we discuss the psychological implications and effects of human-machine interactions and how they may relate to our study. Section 5 is devoted to conclusions and illustrates how this research can contribute to understanding human-machine interaction and to rehabilitation processes activated by "crucial paradigms."

2 Statistical analysis

2.1 Data collection

This study adopts the perspective of analyzing the behavior of a racecar driver as if they were a musician for the reasons discussed in the Introduction. We focus exclusively on their fastest lap during the qualifying session of single championship event of the E4 Formula 4 Championship. To test the utility of the engine-instrument analogy with data, we adopt an approach similar to the study of music and treat racecar engine noise as if it were, in fact, music. For this

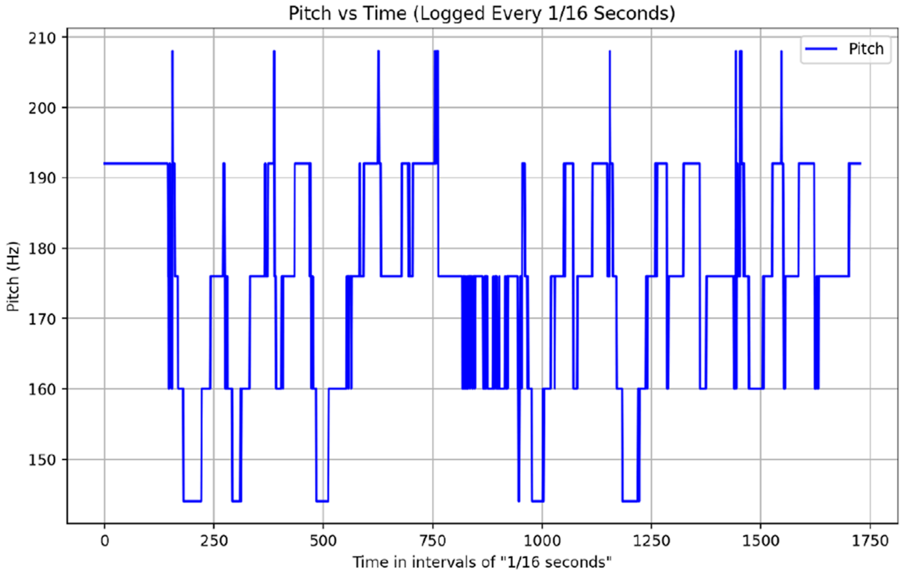

FIGURE 1
Example of Pitch vs. Time making up the pitch-change time series for this driver. This is the driver-engine interaction data from which the scaling index δ is determined for this driver and racecar.

reason, rather than analyzing the "standard" signal of the engine in terms of Revolutions Per Minute (RPM) or the exact points on the track where gear shifts occur, the complex sound of the racecar was sampled as if it were the actual auditory result of a musical instrument.

The technical equipment used consists of an onboard camera (equipped with a microphone) mounted on the racecars. The same camera model (Smarty-Cam 3 GP by AIM Tech S.r.l., data sheets available on request) is used in all racecars and is installed in the same relative position. The data recorded by the camera are saved to digital storage and analyzed after the race using Python programming language to perform the analysis. The logic of the Python script is described next. Upon analyzing the audio portions of the racecar videos, the frequency of engine noise was sampled every 1/16 s. This time interval was chosen to ensure that it is smaller than the fastest gear change by the driver, as the gears are changed by a button on the steering wheel and may occur very rapidly. Note that this time series defined for each driver in the same way constitutes what we call the signal; Figure 1 shows an example of the pitch of the signal versus time in graphic form.

The size of the scaling parameter δ was obtained by processing the engine signals of each of the drivers obtained from the recordings during the same official qualification session conducted from 8:30 a.m. to 8:46 a.m; the official weather conditions were: humidity 74 percent, "dry" condition, Air temperature 22 °C, Track temperature 21 °C [16]. The track name and the race date are not given to guaranty anonymity of the participants.

There are no other specific eligibility requirements, but the standard physiological minimum values of an agonist athlete. Of all entrants, 12 drivers participated in this study. They have ages in the range = 16.25 ± 0.86 (Mean ± Standard Deviation (SD)). Before entering this category, each of them had several years of "carting" experience: 8.83 ± 2.98 (Mean ± Standard Deviation (SD)) years. This refers to experience in a more rudimentary type of vehicle. All drivers were male. Note that the analysis covers every racecar driver who voluntarily participated in the study (authorized by the legal supervisor), without any entry selection to control possible selection or confirmation bias. The identities of the racecar drivers have been kept secret. For this reason, the 12 racecar drivers are referred to as 'A' through 'L.'

It is interesting to note that the examined racecar drivers represent a group of athletes who, despite their young age, are already *de facto* professional racecar drivers. This can be shown beyond their actual performance on the track, from their extensive prior carting experience, and from the lifestyle they report leading. In fact, for example, 10 out of the 12 drivers analyzed (83 percent) state that they attend online school to free up more time every day for racing simulators and gym training specifically aimed at improving the physical traits essential for driving performance.

In the professional motorsports career, Formula 4 is the first category in which drivers can compete after carting. According to the objectives of the International Automobile Federation (FIA), the main purpose of the E4 Formula 4 championship is to serve as a training category (despite the very high level of the athletes), preparation for higher formulas such as Formula 3, 2, and 1, or other advanced series so that it is a category where efforts are made to standardize the racecars as much as possible to highlight the skills of the drivers.

To achieve this, the racecars (named Tatuus T-421) are built, as far as possible, by the same manufacturer to strict technical specifications dictated by a well-defined regulation. This situation differs greatly from other championships, e.g., Formula 1 where each team is free to design and build almost every part of the racecar as the technical side is a fundamental part of challenge. In contrast, in Formula 4, the main goal of the manufacturer is to have the racecars as similar as possible. Of course, there are likely minor differences between individual racecars, but their effects are difficult to quantify, especially in objectively clear terms. To the best of our knowledge, there are no scientific studies on the matter, and this is a limitation of this study, but considering that having identical racecars is a fundamental pillar of the manufacturer and of the organizers, this type of championship appears to offer the most balanced competitive environment in which to address the realistic objectives of this study.

In short, all drivers used the same type of racecar, were on the same race track, and were observed during the same qualifying session using the same camera model and using the very same analysis tool. In essence, our goal was to create an optimal experimental setup that, despite the many limitations and constraints of this study, enables the best achievable and attainable data collection in a real-world racing environment during an official competition.

This study adopts the perspective of analyzing the behavior of a driver as if they were a musician. For this reason, the focus has been placed on the engine sound they produce.

2.2 Finding and analyzing crucial events

Cognition generates an ergodicity breakdown as defined in Section 1.2. Following the earlier work of Allegrini et al. [50], we interpret this effect as the occurrence of crucial events. We define a crucial event as an input into the engine by the driver, whether visible or invisible. Each individual gear shift may or may not be perceived as a crucial event. It is important to note that according to Allegrini et al. [50], crucial events are typically invisible, but fortunately a method has been developed to detect unseen crucial events. The theoretical foundation of the analysis used in this paper is given by the 2001 work [10]. This paper is an approach to the adoption of diffusion entropy analysis discussed in [8, 9] and in the more recent publication [51]. The first paper, which we call Foundation #1 [10], provides a technique to detect invisible crucial events, and the second, which we call Foundation #2 [51], provides an intuitive explanation of why crucial events are invisible. Foundation #2 is a contribution to the study of cell motility with the main purpose of fighting Glioblastoma, a cancerous cell that spreads in the brain. This cell is assumed to adopt a Lévy walk in a two-dimensional reference system. The cell swims with constant velocity along one given direction, and from time to time it changes its swimming direction. The swimming process is modeled as the result of many small jumps of equal value that make the cell move with constant velocity. The projection along either the x- or the y-axis has the effect of changing the intensity of these small jumps, and the invisible change of direction is signaled by the time at which the small visible jumps change intensity to reveal the effect of the direction changes. We investigate the time interval between any two consecutive crucial events to be given by the waiting-time Probability Density Function:

$$\psi(\tau) = (\mu - 1)\,\frac{T^{\mu-1}}{(\tau + T)^{\mu}}, \tag{3}$$

with $1 < \mu < \infty$. Physiological processes correlated with brain dynamics are characterized by the scaling parameter:

$$\delta = \frac{1}{\mu - 1}, \tag{4}$$

which achieves the maximum scaling parameter value of $\delta = 1$ for $\mu \geq 2$, which is usually interpreted as a manifestation of maximum intelligence or "adaptability" [52]. It is the "adaptability" that we measure in the short term, for a reflection of long term success. For a better understanding of what the driver is adapting to, please consider Section 2.1.

We analyze the engine pitch (signal) of racecars using Foundation #1 to detect the invisible crucial events. This makes it possible for us to activate Foundation #2 for the statistical analysis. The time distance between two consecutive gear changes is filled with 1 or −1 at the flip of a fair coin. Consequently, sometimes the fair coin results in subsequent similar realizations. In cases like this, we rely on Foundation #2 to find the crucial events. Foundation #2 is adopted to assign the scaling parameter, δ, to each driver.

In conclusion, as a result of this procedure, we generate a time series of crucial events with the i^{th} event separated by the $(i+1)^{th}$ event by the time length n_i. The work of Grigolini et al. [9] illustrates three different proposals to convert the time series of crucial events into a time series $\xi(t)$ to analyze with the diffusion entropy analysis method. We choose the "velocity model" as our method to generate $\xi(t)$ (the trajectory). Despite the fact that [10] shows that the integration process necessary to evaluate the scaling is done with the rule of making a step ahead when a crucial event occurs, we decided to adopt the velocity model proposal corresponding to filling the laminar regions between two consecutive crucial events with constant velocities of 1 or −1. The reason for this choice is the assumption that the velocity model is more appropriate for the dynamics of racecars. This choice led us to establish a connection with the work of Shah et al. [51]. Figure 2 shows an example of $\xi(t)$ using this strategy.

After successfully assigning 1 or −1 to each region, we then integrate $\xi(t)$ to find the trajectory $x(t)$:

$$x(t) = \int_0^t \xi(t') \quad dt' \tag{5}$$

Figure 3 shows $x(t)$ as given by the integral in Equation 5.

2.3 Diffusion entropy analysis

The method of diffusion entropy analysis was modified with the introduction of "stripes" [10]. This modification is referred to as Modified Diffusion Entropy Analysis (MDEA). Modified diffusion entropy analysis as devised in Allegrini et al. [10] was used to detect temporal complexity within time series data. Modified diffusion entropy analysis first detects crucial events in the time series, defined as the zero crossing times of that time series, and then to process the time series, it assigns the number +1 to those times and 0 otherwise, and finally transfers the sequence of 1s and 0s into the diffusion trajectory by cumulative summation of crucial events (as defined by the sequence of 1s and 0s). The modified diffusion entropy analysis measures the scaling parameter δ of the diffusion process. The evaluated scaling parameter δ is connected to the temporal complexity index μ of the sequence of interevent time intervals τ between such crucial events, where the waiting-time probability density function has an inverse power law which may be obtained from the asymptotic form ($\tau \gg$ T) of Equation 3 with $2 < \mu < 3$ and as mentioned earlier the physiological processes correlated with

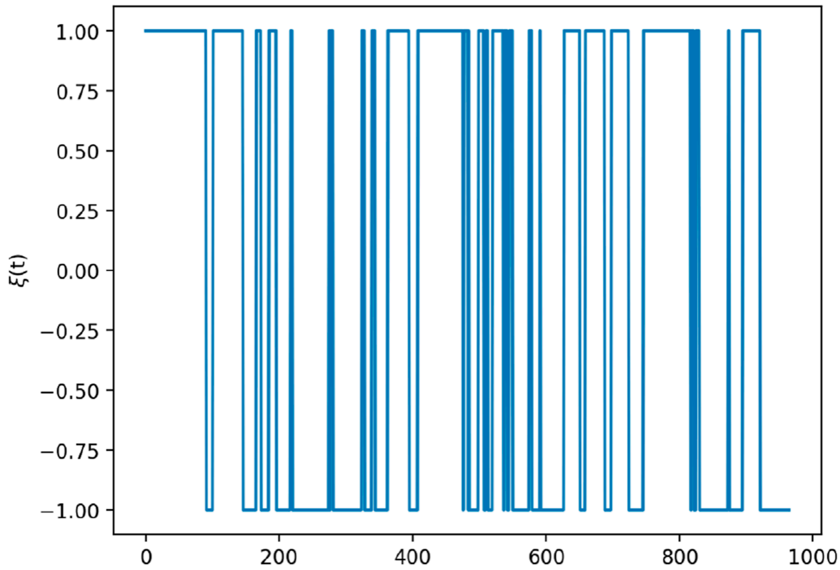

FIGURE 2
1 or −1 for each region between gear shifts.

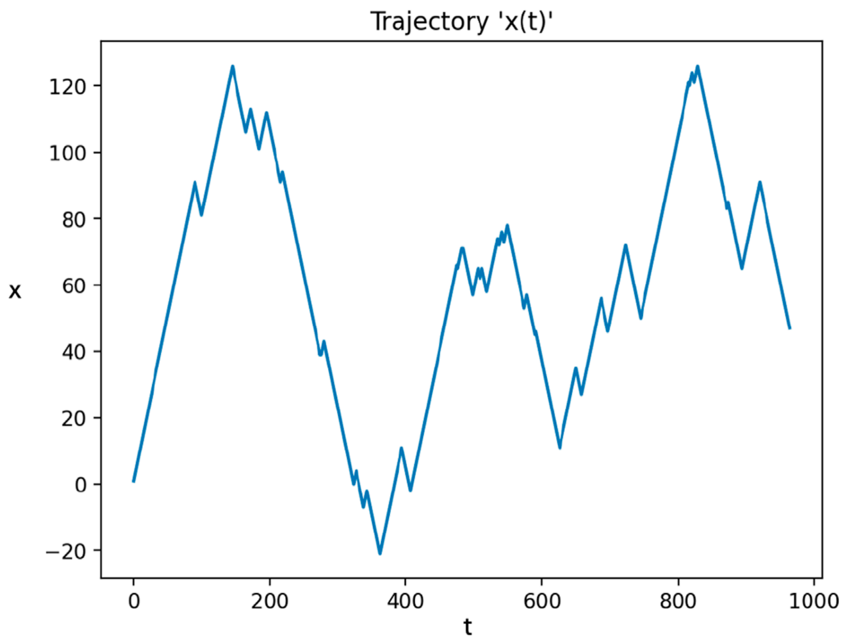

FIGURE 3
Trajectory "$x(t)$".

brain dynamics are characterized by the relation between the scaling indices μ and δ: $\mu = 1 + \frac{1}{\delta}$.

Modified diffusion entropy analysis is used to detect additional crucial events by means of a finer coarse graining. In this new method, to define the crucial events, rather than one threshold (zero-crossing), a number of stripes define the crucial events as the times at which the time series passes from one stripe to another. For further details on the theory and method of modified diffusion entropy analysis, see, for example [53, 54].

Two basic assumptions supporting both modified diffusion entropy analysis and complexity synchronization analysis are that the crucial events extracted from the empirical time series have independent time intervals between crucial events and that these sequential time intervals have an inverse power law probability density function. These assumptions will be tested

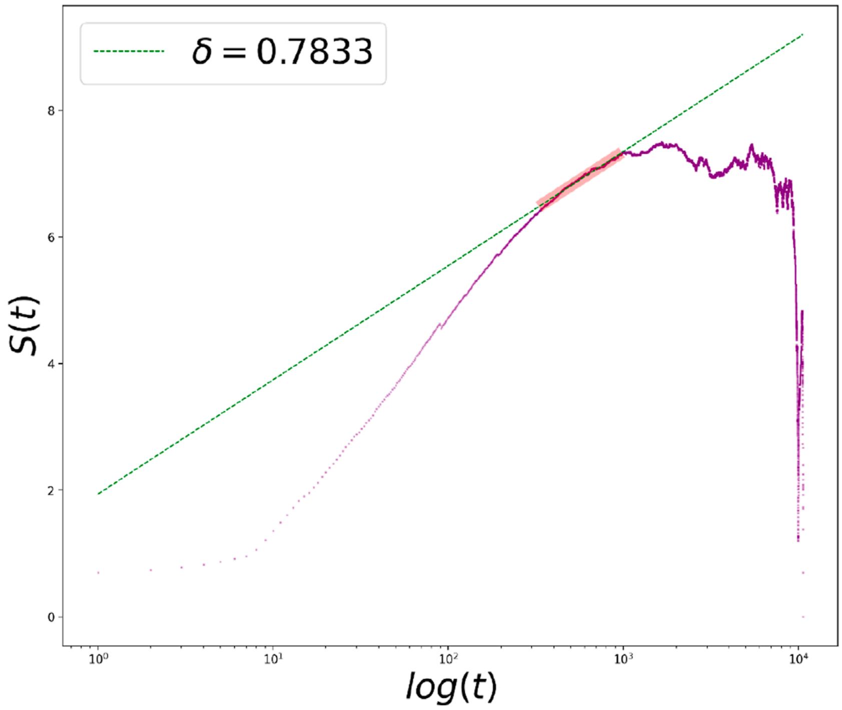

FIGURE 4
Example Shannon Entropy Graph (the δ is determined as the slope of the intermediate asymptotic shown in orange).

when determining the optimal parameters for modified diffusion entropy analysis processing of heterogeneous data in our proposed research. For modified diffusion entropy analysis processing, two important things are important. These are the determination of the best stripe size and the linear fit region of the entropy versus the logarithm of window length plot, see Figure 4 for an example of such a plot. In a recent paper, Schizas et al. [54] discussed the theory and methods of complexity synchronization analysis and how to automate the selection of both parameters for large-scale analyses. Briefly, the systematic variation of the stripe size and compare the empirical distribution of τ's with the theoretical inverse power law probability density function using the Kolmogorov-Smirnoff statistic. The KS statistic quantifies the maximum absolute difference between the inverse power law complementary cumulative distribution function of empirical crucial event time intervals and a candidate theoretical inverse power law probability density function as a function of the stripe size (see Equation 3 in Schizas et al. [54]).

Another option is to verify that the crucial events extracted from the under-sampled signal match a theoretical inverse power law probability density function, and if it does, the results may be valid but would need to be further tested using surrogate data with known properties; see, e.g., [54]. In addition, other measures of complexity that are better suited to shorter time series could also be tested, e.g., detrended fluctuation analysis, 1/f spectra, permutation entropy, etc.).

The method of diffusion entropy analysis [10, 51] is based on the information approach to entropy proposed by Shannon [55]. This leads us to the use of the Gibbs entropy:

$$S(t) = -\int_{-\infty}^{\infty} dx \quad P(x,t) \log P(x,t), \tag{6}$$

where $P(x,t)dx$ is the probability that the integration of the time series $\xi(t)$ generates the PDF that a single walker has the distance x from the origin at time t. To evaluate $P(x,t)$, we should generate a large set of trajectories of the same kind as the trajectory $X(t)$ of Figure 3 that has length "L". All of these trajectories are found at $x = 0$ at time $t = 0$, generating a cloud of increasing size with time.

On the other hand, we only have one trajectory, so we address the problem with the procedure of utilizing a moving window of size l. We assume that the trajectory of Figure 3 has a total length L and divide the trajectory of Figure 3 into L/l trajectories of size l. The window l increases incrementally. Each of these trajectories is shifted in time and space in such a way that the left pair of values for x and t coincides with $x = 0$ and $t = 0$. Unfortunately, L/l is not yet a large enough number to yield an accurate evaluation of $P(x,t)$. For this reason, rather than using L/l non-overlapping trajectories, we use overlapping trajectories. The first trajectory moves from $x = 0$ and $t = 0$ to $x = X(l)$ and $t = l$. The second trajectory moves from $X(1)$ at $t = 1$ to $X(1+l)$ at $t = l+1$. The trajectory n^{th} moves from $x = X(n)$ at time $t = n$ to $x = X(n+l)$ at time $t = l+n$. Of course, there is an upper limit to the number of different trajectories, due to the fact that we cannot use trajectories with a right border at values x larger than the length L of the trajectory of Figure 3. The empirical trajectory provides the same scaling δ whether 1 or -1 alternate or are assigned at the flip of a fair coin. As the window size increases, the number of sub-trajectories decreases.

The values found in this manner are then weighed against each other to find the probability of any given value $P(x,t)dx$. The probability density function $P(x,t)$ is entered into the Shannon entropy formula $S(t)$ given by Equation 6, for every $P(x,t)$. The probability density function may be described as follows, according to the renormalization group approach [6, 7]:

$$P(x,t) = \frac{1}{t^{\delta}} F\left(\frac{x}{t^{\delta}}\right). \tag{7}$$

Inserting Equation 7 into Equation 6 yields the following equation, where "A" is a constant reference Shannon entropy:

$$S(t) = A + \delta \log(t). \tag{8}$$

This approach enables us to find the intermediate asymptotic of the Shannon entropy graph, $S(t)$, Equation 8 allowing us to calculate δ [6, 7]. Figure 4 shows an example of a Shannon entropy graph.

2.4 Championship results

The actual scaling value δ has been evaluated for a single lap of the track (see Section 1, Section 2, Section 3). To investigate the potential significance of δ, this parameter was correlated with two objective performance indicators. Specifically, the analysis considered the final standings of the qualifying session whose engine noise is analyzed in this study and the final ranking of the entire E4 Formula 4 Championship (a global parameter consisting of nine races) each driver accrued by the end of the championship.

It is interesting that the final results of the qualifying session exhibit a statistically significant Pearson linear correlation with δ, yielding $r = -.86$ ($p < 0.001$) as recorded in Table 1. Moreover, it should be noted that the final ranking of the entire E4 Formula 4 Championship shows Pearson linear correlation parameters that exhibit a strong statistically significant correlation with δ being $r = -.90$ ($p < 0.001$). It is important to note that the results of the championship statistically compensate for lucky and unlucky events and more accurately reflect the skill of the racecar driver. These findings suggest that higher scaling δ is associated with better overall performance of the racecar driver.

As a further finding of this work, another racecar driver (referred to as driver "X") has been studied in addition to drivers "A" through "L". Driver "X" drove in the same championship and performed the same qualifying laps as racecar drivers had done, but some years earlier in a previous edition of the same championship. It has been considered because driver "X" currently competes in top-tier racing categories being regarded as one of the best drivers in the world. The scaling δ of driver "X" has been analyzed in the same way as the other racecar drivers of this work, finding $\delta = .98$, higher than the best found among the 12 other racecar drivers analyzed. This result can be viewed as consistent with the interpretation of this work. A high degree of accuracy for the scaling parameter δ is chosen because the values can be very similar.

TABLE 1 Comparison of δ and qualifying lap standings.

Driver	Qualifying lap rank	δ
A	1	0.9735
B	2	0.9374
G	3	0.8314
C	4	0.8968
D	7	0.8519
E	10	0.8441
H	11	0.8112
F	14	0.8318
J	17	0.7515
K	19	0.7452
I	24	0.7833
L	25	0.6274
X	Didn't race in championship	0.9801

3 Training process

3.1 Anonymous driver #1

Table 1 shows the relationship between driver performance (time of fastest qualifying lap) and δ calculated as described in Section 2.3. This relationship reveals that a higher δ for the fastest qualifying lap is associated with a better overall driver performance in their championship.

A similar comparison can be made with a single racecar driver training over many practice laps. A different racecar driver, who will remain anonymous and therefore named "AD1" (a novice driver that has not yet been studied in the earlier sections of this paper), was analyzed for δ of each lap they drove in a practice session. This racecar driver drove 20 laps in succession on "Day 1" using the same type of racecar that was used by racecar drivers "A" through "L." These practice sessions are referred to as "long runs." The racecar driver slowly drives the course for laps 1 and 2 to warm the tires and the engine, and then "pushes" for the remaining laps. The term used is "warm-warm-push." This driver is also alone on the track (drivers maneuver the course at great distances from one another) and has a great deal of freedom. This provides an opportunity to view them at their maximal "adaptability." However, external factors can inhibit the measurement of their full potential. Each lap time for "day 1" is weighed against the δ computed for that lap. This is illustrated in Figure 5.

The trend in Figure 5 is similar to that in Table 1 (excluding laps 18 & 19). The exact values are shown in Table 2.

During lap 11, AD1 caught up with a different driver who previously started the practice session because of an initial different

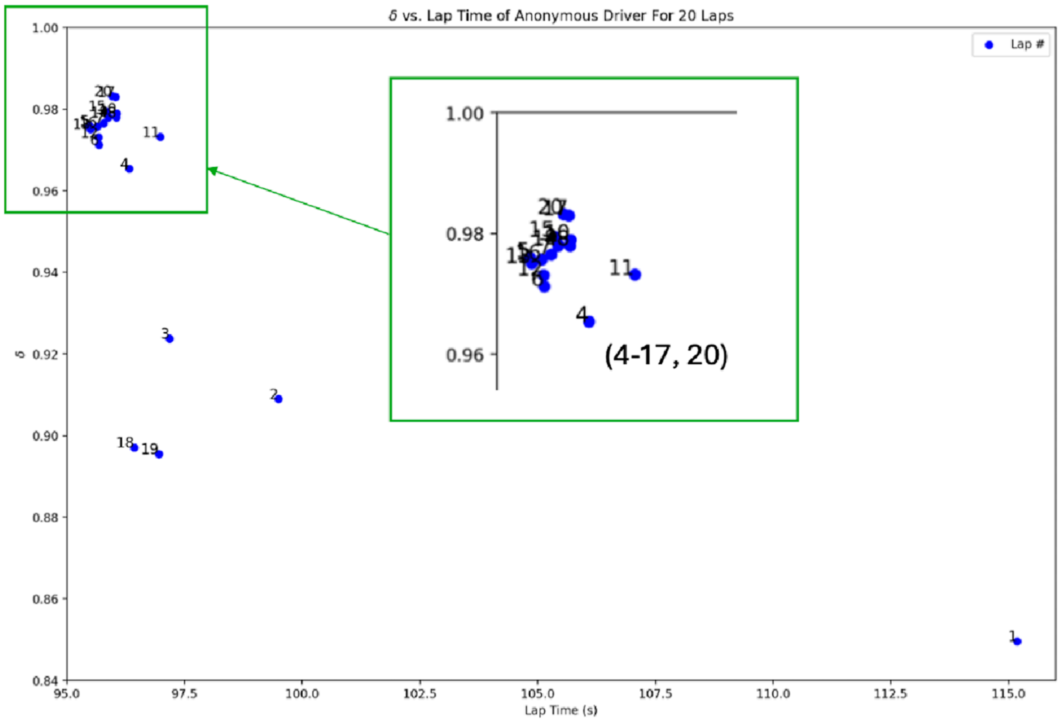

FIGURE 5
AD1, day 1 δ vs. lap time. Laps 4–17 & 20 are very close, thus there is a portion zoomed for better visibility.

driving pace. During laps 18 & 19, the distance between AD1 and the car before him became very close. This caused a "difficulty" which is apparent in the measurement of δ. In fact, the two cars engaged in a sort of "racing behavior" in terms of a "one-on-one fight", with evident effects. For example, on lap 18, AD1 hit the brakes abruptly to avoid the car ahead experiencing what is called a "locked up" tire. This is very risky because it can lead to a "flat spot" on the tire. During lap 19, AD1 had to avoid the same car because that car suddenly used the brakes and experienced a "lock up." The phenomenon of scaling δ lowering during apparent challenges in laps 18 & 19 is in line with the hypothesis of Correll [56, 57]. Correll states that $1/f$ noise tends towards white noise (white noise corresponds to $\delta = 0.5$) in the presence of a difficult task. These difficult tasks affect adaptability in the way described in [58, 59]. The authors of [57] found that the transition to white noise from $1/f$ noise is due to the modified neuron interaction generated by the goal of settling a difficult task. This is an important issue that leads us to establish a connection with the open issue of cognition [21, 60]. This is in line with the approach of Heidegger [61], adopted by Dotov [62, 63]. The Psychological experiments conducted by Dotov are interpreted as a decrease of the scaling δ due to a difficult task. According to [57], this generates the transition to white noise for extremely difficult tasks. We believe that the small decreases of δ are a sign of the same process as that of [56, 57, 62, 63].

AD1 also drove a new training session consisting of 8 laps on "day 2." It is notable that AD1 did not encounter any other racecars during this run thus there are no significant deviations from the "learning curve" of the laps. This is also in line with Heidegger [61–63]. Figure 6 shows a graph of δ versus lap time for AD1 on "day 2" and Table 3 shows the exact values.

Figure 6 represents a clear improvement from "day 1" to "day 2." It is important to stress that external factors, such as weather, humidity, temperature, and barometric pressure, can play a significant role in lap times and driver performance. However, the scaling index δ can be used as a tool to help analyze the "adaptability" of the driver during that lap. Although δ is not an absolute measurement of success, it seems to correlate quite well with driver performance.

3.2 Anonymous driver #1 vs. anonymous driver #2

A second anonymous racecar driver, named "AD2" (more experienced than AD1), was analyzed in the same manner. AD1 and AD2 drove independently on "day 2" and were compared against each other.

In numerical terms (please see Table 4), the greater experience of AD2 is evident in the mean lap-time value (AD#2 is more than 1 s lower than AD#1) and δ moves accordingly and consistently. Interestingly, δ expresses a Standard Deviation one order of magnitude lower in AD2 (whereas lap-time standard deviations are similar), showing a much more constant behavior in the more experienced driver.

The higher experience of AD2 is apparent in Figure 7 and Table 4 when comparing the scaling δ with the lap times.

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TABLE 2 AD1: Comparison of day 1 δ (scaling indices) & lap times in order of lap driven.

Lap	Lap times (s)	δ
1	115.18	0.8496
2	99.49	0.909
3	97.18	0.9238
4	96.32	0.9654
5	95.47	0.976
6	95.68	0.9713
7	95.78	0.9766
8	96.05	0.978
9	95.88	0.9782
10	96.06	0.979
11	96.98	0.9732
12	95.67	0.9731
13	95.5	0.9751
14	95.88	0.978
15	95.83	0.9795
16	95.64	0.9757
17	96.03	0.983
18	96.43	0.8971
19	96.95	0.8955
20	95.96	0.9832

AD2 exhibits behavior that is also in line with Correll [56, 57] and Heidegger [61–63] as the trend shows in Table 5. AD2 has much more learning experience and, therefore, knows how to adapt more efficiently than a novice driver with less experience.

4 Model of motor skill learning

Organized auto racing may be conceived as a sociocultural activity that requires highly advanced perceptuomotor skills of racecar drivers, in which spatiotemporal relationships between the performer and the performance environment continually interact while exchanging energy, matter and information [64]. More broadly, from a motor learning and control perspective, these results may be interpreted from the non-equilibrium model of motor learning [64, 65].

Compared to the average brain development of a novice racecar driver, professional Formula drivers showed a smaller volume recruitment of the sensorimotor, parietal, and prefrontal regions, stronger connections among these regions, and greater integration of information, as reflected by a higher temporal variability of the signal during motor reaction and visuospatial tasks [39]. These findings suggest ‘increased efficiency in attentional and sensory information processing along with reduced resource consumption in racecar drivers, as well as a greater ability to adapt to rapid changes in environmental demands. These findings are consistent with the non-equilibrium model of motor learning.

In a longitudinal learning study, Sultana et al. [66] examined novice racecar drivers in a simulator over the course of 10 training sessions while recording their electroencephalograms (EEG’s). They observed decreases in theta (4–8 Hz) band power across nine regions of interest during the 10 training sessions and a positive correlation between theta power and lap times, suggesting a decreasing need for high-level cognitive control as skills become more automated. They also observed increases in effective connectivity between frontocentral and occipital regions in the alpha band (8–12 Hz) during training sessions, suggesting ‘greater functional coordination between motor planning and visual processing areas as they adapted to the racing task. In general, the authors concluded that the general mechanistic principle underlying learning was increased efficiency enabled by the plasticity of cognitive processing and visuomotor coordination.

Most neuroimaging studies on racecar drivers have used simulators or passive viewing of races (see [38] for a review). In a unique real-world racing study Rito Lime et al. [34] conducted a case study on a Formula E Champion driving on a race track under extreme conditions (high speed, low visibility, low temperature, wet track) while recording electroencephalograms and eye and body kinematics of the racecar driver. They found positive correlations for the acceleration and rotation of the hands with the alpha and beta powers, and a negative correlation with the delta power. Alpha and beta power increases preceded steering movements by 100 ms, while delta power decreases were synchronized with steering movements. They also showed that during straight segments of the track, there were no correlations between steering movements and electroencephalogram spectra, but during curved segments, the delta power decreased while the alpha and beta power increased. This study by [34] showed for the first time interactions between the neural and behavioral systems of a racing champion under extreme driving conditions.

According to the non-equilibrium model of motor learning, two cyclic processes constantly interact during the course of learning and performing motor skills. The first process, functional stabilization, is the emergence of a motor pattern whose spatio-temporal structure reconciles order and disorder. The second process, adaptation, results in a growing complexity of the hierarchical organization of neurophysiological and neuromuscular systems, which facilitates self-organization, flexibility, and adaptability of the functionally stable system to perturbations or challenges [64]. As hierarchically organized systems, macro and microstructural levels are conceived in the non-equilibrium model of motor learning model [64]. The macrostructural level reflects a general spatio-temporal configuration of a task-specific motor skill, which emerges from the interaction among components of that motor skill and is constrained by the coupling of intention and task specificity (e.g., navigating a racecourse faster than opponents

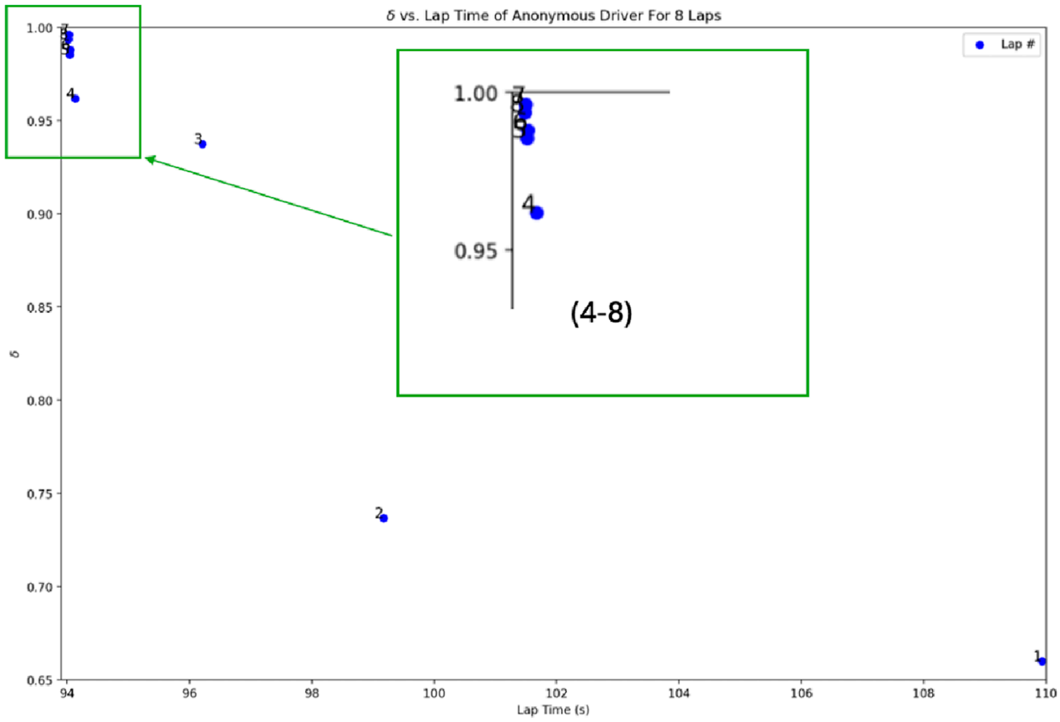

FIGURE 6
AD1, day 2 δ vs. lap time. Laps 4–8 are very close, thus there is a portion zoomed for better visibility.

TABLE 3 AD1: comparison of day 2 δ (scaling indices) & lap times in order of lap driven.

Lap	Lap times (s)	δ
1	109.93	0.6602
2	99.17	0.7369
3	96.21	0.9377
4	94.13	0.962
5	94.04	0.9856
6	94.05	0.9881
7	94.03	0.9962
8	94.02	0.9937

TABLE 5 AD2: comparison of day 1 δ(scaling indices) & lap times in order of lap driven.

Lap	Lap times (s)	δ
1	106.93	0.7429
2	99.24	0.985
3	96.21	0.9887
4	94.13	0.9892
5	94.04	0.9903
6	94.05	0.9868
7	94.03	0.9912
8	94.02	0.9928

TABLE 4 The mean and standard deviation of δ and lap time for AD1 and AD2.

	AD#1 lap time	AD#1 δ	AD#2 lap time	AD#2 δ
Mean	95.565	0.977	94.413	0.989
SD	0.838	0.022	0.881	0.002

while avoiding collisions and accidents). The microstructural level, on the other hand, refers to the specific components of that motor skill (e.g., accelerating/decelerating, steering, shifting gears). These two component processes continuously interact such that they co-exist as complementary or cooperative functions from which spontaneous behavior of the whole emerges.

The learning of motor skills, then, proceeds along a continuum that involves cycles of functional stabilization and adaptation [64].

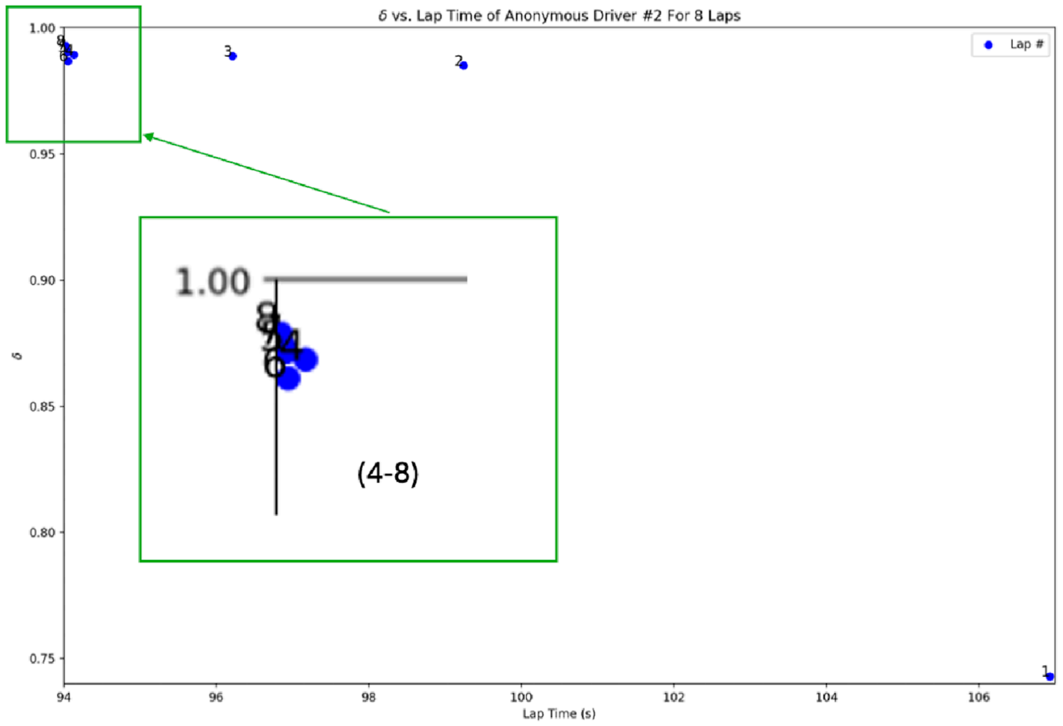

FIGURE 7
AD2, day 1 δ vs. lap time. Laps 4–8 are very close, thus there is a portion zoomed for better visibility.

The greater the opportunity to experience perturbations to the functional stability of a motor skill (i.e., its macrostructure), the greater the opportunity for that pattern to reorganize itself (via its microstructure), and the greater the complexity of the hierarchical organization of neurophysiological and neuromuscular systems, and the greater the refinement and improvement of the motor skill. Perturbations may come from both internal sources (perceptual, cognitive, and affective states) and external sources (task and environment). When such perturbations occur, positive feedback mechanisms function to amplify the discrepancy, and through adaptation a new stability regime may emerge that takes order from disorder [64]. The ability to gain from disorder was shown by [67] to be a new kind of complex interaction which he named antifragile.

In the context of the present study, the drivers are relatively young and may be considered to be in early stages of learning (exhibiting a multi-stable regime), and individual differences in skills among drivers may be quite variable. In addition, individuals may vary considerably in cognitive and affective state/trait variables relating to competitiveness, arousal, anxiety, coping mechanisms, pre-competition preparations, etc. According to the Hanin individual zones of optimal functioning model [68], the performance of athletes is best when they're in their individually optimal zone of functioning. Thus, intra-individual differences may also be considerable from one performance to the next, depending on variations in pre-competition readiness. With these considerations in mind, it is conceivable that the negative correlation observed in this study between complexity scaling indices and race performance (higher scaling associated with a lower numeric rank, indicating higher level performance) reflects the more advanced adaptation of more successful drivers who exhibit greater complexity in the hierarchical organization of the neurophysiological and neuromuscular systems that underlie performance.

From a broader ecological perspective [69, 70], the driver and the racecar are merged into one complex system, along with the environment, and cognition is embodied and distributed beyond the body to the racecar (i.e., driver-racecar unit [36, 37]). Embodied models of motor learning [24, 70, 71] view the mind, body, and environment as continuously mutually influencing each other and shaping the emergence of behavior. The neural, physiological and environmental systems are all informationally, energetically and mechanically coupled, and movement patterns emerge during the performance of the task depending on the specific constraints on the performer, the task, and the environment (based on the constraints, the advantages provide opportunities or invitations for actions [70]). There exists a paradoxical relationship between stability and variability, in which the performer seeks to consistently repeat a performance outcome, although the movement pattern used to achieve this outcome varies from performance to performance. The transition between the stable and unstable phases occurs through self-organizing processes that facilitate the learning of motor skills.

5 Conclusion

These findings show that higher scaling δ is associated with better overall driver performance over a long period of time. This result is consistent with the literature on music and brain effort. In particular, previous studies exploring the relationship between music and cognition have shown that a computer exhibits a lower scaling value δ, while humans display higher scaling values [27]. The results obtained in this experiment may lead to interesting interpretative hypotheses. The most straightforward conjecture suggests that as the δ value increases, it corresponds to an enhancement of a skill parameter related to cognitive ability, as this is the primary factor distinguishing a computer from a "human musician." Essentially, better driving performance can be influenced by human-specific characteristics possibly linked to increased cognitive effort.

It is important to note that a limitation of this study is the low number of racecar drivers analyzed. However, the high quality of the results obtained, along with the consistency of our theoretical results with the empirical findings of other investigators, is significant. The very strong correlation found and the case of "Driver X," pose these results as a very promising research direction. Moreover, the intuitive study of anonymous drivers AD1 and AD2 shows a consistent behavior that possibly makes this research direction even more worthy of a deeper consideration. This work is preliminary and further studies need to be done; however, it is very interesting that a single parameter δ could capture the behavior of the driver's mind. We aim to investigate the potential implications for the therapeutic and cognitive performance of CERT [72]. This is related to the work of [45, 48].

Finally, we want to stress that this paper may favor a debate on the open issue of cognition [21, 22] and the challenging issue of how it may relate to machine learning. The analogy between music and racecars has its foundation in the complexity matching effect. Its quantum mechanical origin seems to be very appropriate to establish a bridge between Tononi [21] and Faggin [22].

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Ethics statement

The studies involving humans were approved by University of North Texas Internal Review Board. The studies were conducted in accordance with the local legislation and institutional requirements. The participants provided their written informed consent to participate in this study. Written informed consent was obtained from the individual(s) for the publication of any potentially identifiable images or data included in this article.

Author contributions

JS: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review and editing, Funding acquisition, Project administration, Resources, Supervision. YS: Conceptualization, Investigation, Supervision, Visualization, Writing – original draft, Writing – review and editing. LT: Conceptualization, Data curation, Resources, Supervision, Writing – review and editing. GC: Conceptualization, Data curation, Resources, Writing – review and editing. RG: Conceptualization, Data curation, Resources, Writing – review and editing. SK: Conceptualization, Funding acquisition, Supervision, Writing – review and editing. PG: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review and editing. BW: Conceptualization, Methodology, Supervision, Writing – review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Publisher's note

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