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Optimizing the home-school-society collaborative mechanism for college students' mental health: an evolutionary game theory approach

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The mental well-being of college students is a critical concern that requires a collaborative effort from families, schools, and society. However, the internal mechanisms and key factors that drive effective collaboration among these three parties remain unclear, hindering the establishment of a robust support system. This study constructs a tripartite evolutionary game theory model involving families, schools, and societal institutions. The model investigates the impact of government policy support (including financial subsidies), cost-sharing arrangements, and benefit allocation mechanisms on the behavioral strategies of all stakeholders. Stability analysis and numerical simulations were employed to examine the dynamic evolution of the system. The simulation results indicate that targeted government subsidies significantly enhance the participation enthusiasm and sustained engagement of families and schools. Furthermore, the incentive intensity for social institutions is a pivotal factor in promoting their willingness to open and share resources. A stable, high-collaboration state can be achieved when families receive sufficient policy support, schools' coordination costs are covered, and the benefits for social institutions exceed their input costs. The findings confirm that an optimized incentive structure is essential for sustainable collaboration. This paper concludes by proposing concrete policy recommendations to improve the long-term home-school-society collaborative mechanism for college students' mental health, offering a valuable theoretical and practical framework for stakeholders.

KEYWORDS

college students' mental health, home-school-society collaboration, evolutionary game, government regulation, collaboration mechanism, policy optimization

1 Introduction

In recent years, the prevalence of mental health issues among college students in China has surged, reflecting the impact of social transformations and higher education reforms. The [World Health Organization \(2021\)](https://www.who.int/publications-detail/11-psychological-problems) reports a rising annual detection rate of psychological problems, such as depression and anxiety, which compromise students' academic performance and overall wellbeing, posing challenges to national

talent development and social stability (Ayiro et al., 2023). Thus, creating a supportive educational environment that promotes mental health through collaboration among families, schools, and society has become crucial.

Research increasingly emphasizes the importance of collaborative mental health education. Liu and Xiao highlight this need in primary and secondary education (Liu and Xiao, 2023), while Tang (2023) expands the concept to colleges by proposing a long-term home-school-community mechanism. Theoretical insights by Cavioni et al. (2020) emphasize the role of social and emotional learning as well as resilience building in schools. However, challenges persist, as Wang discusses unclear roles and poor coordination in his “Trinity” model for mental health education (Chu and Wei, 2024).

Innovative strategies are being explored to enhance mental health outcomes. Li advocates for positive psychology methods (Luyu, 2025), and Chen present three-dimensional education models aimed at fostering optimism and resilience (Chen et al., 2023). Nonetheless, barriers such as insufficient staffing, lack of policy support, and reliance on outdated methods hinder progress (Fengxian, 2024). These issues are exacerbated in higher education, where academic competition increases mental health stresses.

Collaborative approaches show promise in addressing these challenges. Research by Zhang et al. (2022) indicates that students’ perceptions of academic value and social dynamics significantly influence their engagement. Strategies like home-school collaboration (Liu, 2025) and unified management practices (Xiaoli, 2024) highlight the effectiveness of coordinated efforts in alleviating anxiety, particularly for disadvantaged students.

Despite these advancements, a cohesive, system-level strategy integrating families, schools, and society remains essential. This study aims to optimize the Home-School-Society collaborative mechanism to improve college students’ mental health by aligning stakeholders, enhancing policy support, and incorporating innovative educational models. The paper will first review pertinent literature, outline methodology, present findings, and conclude with insights and recommendations for future research and practice.

In summary, while there is growing recognition of the importance of collaborative strategies in mental health education, significant barriers still restrict effective integration among families, schools, and communities. This study aspires to address these gaps, contributing to a more supportive environment for students and improving mental health strategies within the broader social context.

1.1 Research questions

This study investigates the core challenges in constructing a collaborative home-school-community education system for university students’ mental health. Specifically, it addresses the following key questions:

1. Driving mechanisms: What internal mechanisms and critical factors facilitate or hinder effective, sustainable collaboration among families, universities, and societal stakeholders? How do these parties make decisions within cooperative frameworks?
2. Cost-benefit dynamics: How can collaborative cost-sharing and benefit-distribution mechanisms be optimally designed? What level of incentive is required to motivate social institutions to openly share resources while ensuring their returns—both tangible and intangible—exceed input costs?
3. System stability conditions: Under what conditions can a stable, highly collaborative tripartite model be achieved? Key considerations include sufficient policy support for families, compensation for universities’ coordination costs, and guaranteed benefits for social organizations.
4. Policy pathways: What concrete, operational policy recommendations can support the development of a sustainable, long-term collaborative mental health education mechanism?

1.2 Methodology

Figure 1 provides a schematic overview of the full research approach for this study presents a theoretical exploration aimed at modeling the underlying mechanisms of the “university-family-community” tripartite collaborative education system through formal theoretical modeling, rather than empirical validation using questionnaires or interviews.

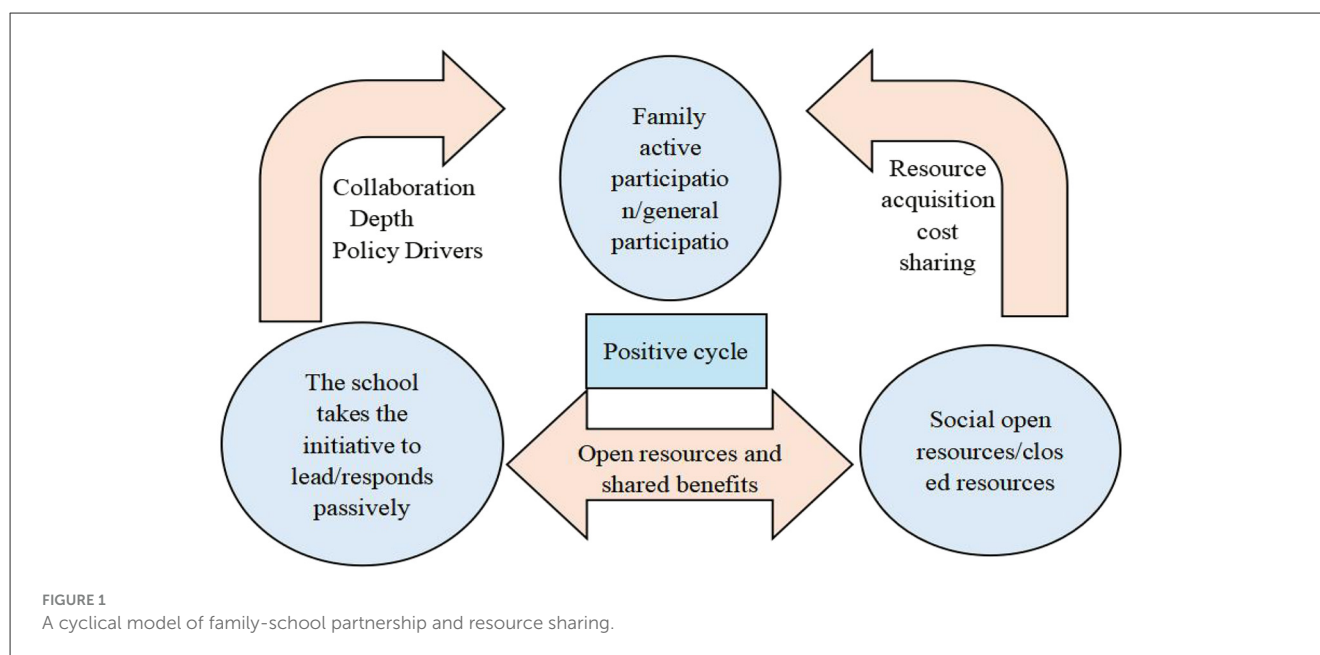
The research design is anchored in two foundational theoretical frameworks:

Ecological systems theory (Urie, 1979): We employ this framework to conceptualize student psychological development as emerging from interactions within microsystems (higher education institutions, families, and communities), mesosystems (linkages among these entities), and exosystems (e.g., governmental policy influences). The evolutionary game model operationalizes this structure by treating each party as an autonomous agent within the microsystem, while explicitly modeling inter-agent interactions at the mesosystem level. External factors, such as policy interventions, are incorporated as exogenous parameters that alter incentive structures, thereby influencing system evolution. This alignment ensures the model’s structural coherence with real-world educational ecosystems.

Social learning theory (Albert, 1977): The behavioral dynamics of the model—specifically the replicator dynamics equation—are grounded in Social Learning Theory. Actors (universities, families, communities) are modeled as boundedly rational agents who adapt their strategies through observational learning and imitation of successful behaviors, based on perceived payoffs (fitness). This provides micromechanistic explanatory power for strategy adoption and behavioral evolution within the system.

Modeling approach: Using evolutionary game theory as our formal analytical tool, we implement a five-step modeling procedure:

- (1) Delineation of the three strategic agents and their action sets;
- (2) Construction of a payoff matrix reflecting costs, benefits, and risks derived from empirical literature and contextual analysis;
- (3) Specification of replicator dynamics equations to capture strategic evolution over time;



- (4) Identification and stability analysis of evolutionarily stable strategies (ESS);
- (5) Numerical simulation of evolutionary trajectories under varying initial conditions and policy parameters.

1.3 Innovation and theoretical contribution

The adoption of evolutionary game theory constitutes a significant methodological innovation in studying collaborative education mechanisms. It enables the translation of complex social interactions into a quantifiable and dynamically simulatable framework, overcoming limitations inherent in purely qualitative approaches.

- (1) From static to dynamic analysis: Unlike traditional cross-sectional studies, our model captures longitudinal dynamics and path dependencies, simulating how systems evolve from fragmentation to cooperation under specific conditions.
- (2) Mechanistic insight into collaboration barriers: The model formally demonstrates how misaligned incentives—such as free-riding behaviors, institutional inertia, and weak community engagement—create social dilemmas (e.g., prisoner's dilemma dynamics). This quantitative framing enables the design of incentive-compatible mechanisms to realign interests.
- (3) Policy simulation for governance innovation: By enabling *in silico* testing of policy interventions (e.g., subsidies, accountability mechanisms, reputation systems), the approach supports evidence-based governance, allowing stakeholders to anticipate outcomes and design robust, context-sensitive strategies.

This research provides a theoretically rigorous and computationally tractable framework for addressing persistent collaboration challenges in educational ecosystems, offering both

conceptual advances and practical utility for policymakers and institutional leaders.

2 Establishment of the tripartite evolutionary game model

2.1 Basic assumptions and model construction

In the context of collaborative mental health education for college students, the three main participants are families, schools, and society, whose interactive behaviors form a game system. It is assumed that the strategies of families include “Active Participation” and “Passive Participation”; those of schools include “Proactive Leadership” and “Reactive Response”; and those of society include “Open Resources” and “Closed Resources.” The symbols and meanings related to the model are shown in Table 1.

College students are in a critical period of psychological maturity and social transition (Koenig et al., 2025), facing multiple challenges such as academic advancement, career planning, and social adaptation. There are also some students who advocate individualism, prioritize personal interests above all else, ignore the interests of others (Williams et al., 2025), lack a sense of social responsibility, and have prominent interpersonal relationship problems (Weibull, 1997). On the other hand, although college students are adults, their mental faculties are not yet mature, and their perception of problems is often highly subjective, arbitrary, and one-sided (Gintis, 2017). Their mental health issues are more complex and latent, urgently requiring deep participation and dynamic collaboration among families, schools, and society. Research shows that young people obtaining health information through social media helps reduce stress (Sandholm, 2020). Through strategic interactions and resource integration akin to evolutionary games, a precise and systematic educational support

TABLE 1 Explanation of game-related parameters.

Parameter symbol	Meaning
x	Proportion of families choosing “active participation in mental health education” ($0 \leq x \leq 1$, $1-x$ for “passive participation”)
y	Proportion of schools choosing “proactive leadership in mental health education” ($0 \leq y \leq 1$, $1-y$ for “reactive response”)
z	Proportion of society choosing “open mental health resources” ($0 \leq z \leq 1$, $1-z$ for “closed resources”)
$R_{s,ij}^s$	Mental health service revenue when schools collaborate with social organizations (i: school strategy; j: social strategy)
α	Government subsidy ratio for schools’ mental health education
S_m	Maximum government subsidy amount for schools’ mental health education
k_{ij}^O, k_{ij}^C	Risk costs for schools to address mental health issues when society opens/closes resources ($\lambda_3 = 0 < k_{ij}^C$)
R_{DC}^h, C_{DC}^h	Revenue and cost of “deep collaboration” between families and schools
R_{BC}^h, C_{BC}^h	The benefits and costs of “basic collaboration” between home and school
β	The government’s subsidy intensity for the home-school collaboration mechanism
S_p	The maximum subsidy amount of the home-school collaboration mechanism
R_A^c, R_p^c	The benefits of cooperation between social professional institutions and schools (under the active/passive strategy of schools)
c_c	The service costs of social professional institutions
θ	The incentive ratio of the government to social professional institutions
R_C^h, R_C^s, R_C^c	The benefits of collaboration among families, schools, and society
$\lambda_h, \lambda_s, \lambda_c$	The proportion of income distribution among families, schools, and society
R_s	The intensity of support from social open resources
c_f^j	The cost of psychological education for families under social strategy j (j: Open/Closed Resources)
k_{ij}	Strategy combination left parentheses i.,j The probability of psychological risk occurrence under the right parentheses (i. : School strategy; j: Social Strategy)
ΔU	The difference between the total benefit of collaboration and the benefit of unilateral actions ($\Delta U = U_C - U_N$)
U_C, U_N	The expected psychological benefits of the collaborative state and the non-collaborative state
f, g, h	The adjustment rates of family, school, and social strategies
s_f	The maximum policy support subsidy that families can obtain

system can be constructed to effectively promote their mental health and all-round development. Therefore, there is an urgent and necessary demand for a home-school-society collaborative education mechanism.

Assumption 1: The main game players in this study include families, schools, and society. Families choose the “active

participation in mental health education” strategy at a proportion of x ($0 \leq x \leq 1$) and the “passive participation” strategy at $(1-x)$; schools choose the “proactive leadership in mental health education” strategy at y ($0 \leq y \leq 1$) and the “reactive response” strategy at $(1-y)$; society chooses the “open mental health resources” strategy at z ($0 \leq z \leq 1$) and the “closed resources” strategy at $(1-z)$. x, y, z are functions of time, reflecting the dynamic evolution of the three parties in mental health collaboration. Their strategy choices are influenced by factors such as mental health benefits, collaboration costs, and risk aversion. Although the core analysis of this model focuses on quantifiable policy and economic variables such as fiscal subsidies (e.g., $\alpha S_m, \beta S_p, \theta R_c$), cooperation costs (e.g., C_{DC}^h, C_{BC}^h, c_c , and profit distribution ($\lambda_h, \lambda_s, \lambda_d$), we fully recognize the important role of complex social-psychological factors such as cultural cognition (e.g., the degree of attention families pay to mental health education), social norms (e.g., the social expectation that schools play a leading role), and psychological expectations (e.g., the belief of social institutions in the long-term benefits of cooperation) in strategy selection. Although these non-quantifiable factors are difficult to be directly set as independent parameters in the model, their influence mechanisms can also be reflected indirectly in the differences of subjects’ “perception” of expected profits. Specifically, the actual decision-making of subjects is not only based on the objective payoff set in the model but also driven by their subjective perceived payoff.” The perceived payoff is adjusted by the above-mentioned non-quantifiable factors and can be formally expressed as: $\tilde{R}_{ij}^k = \delta_k R_{ij}^k + \xi_k, k \in \{f, s, c\}$ among them: R_{ij}^k the objective payoff (e.g., family payoff R_{ij}^f , school payoff R_{ij}^s , social payoff R_{ij}^c) calculated based on quantifiable variables and defined in Model Tables 1, 2.

$\tilde{R}_{ij}^k$ represents the perceived payoff of subject k (family f , school s , society C) under a specific strategy combination (i, j) , and it is the direct basis for its strategy selection.

δ_k (Perception Adjustment Factor): It is a coefficient greater than 0, used to depict the magnification or reduction effect of non-quantifiable factors on the objective payoff.

$\delta_k > 1$: It means that positive cultural cognition, favorable social norms, or optimistic psychological expectations magnify the subject’s perception of the objective payoff (for example, families highly recognize the value of mental health and believe that the same fiscal subsidy αS_f brings higher utility).

$\delta_k < 1$: It means that negative cultural cognition (e.g., the concept of “valuing academic performance over psychology”), unfavorable social norms, or pessimistic psychological expectations weaken the subject’s perception of the objective pay off (for example, schools believe that taking on a leading role faces excessive social pressure or reputation risks, and even if there is a subsidy βS_p , its perceived net payoff is relatively low).

$\delta_k = 1$: It means that the perceived payoff is equal to the objective payoff, and non-quantifiable factors have no adjustment effect (ideal neutral state).

ξ_k (Perception Deviation Term): It is a constant term (can be positive or negative), representing the fixed psychological utility or cost generated by non-quantifiable factors and independent of the objective payoff (for example, the improvement of the brand image brought by social institutions fulfilling social responsibilities $\xi_c >$

0, or the psychological burden brought by family participation in cooperation ($\xi f < 0$).

Assumption 2: When schools choose to collaborate with social organizations, their mental health service revenue is R_{ij}^s , ($i \in \{A, P\}$) denoting proactive or reactive strategies; $j \in \{O, C\}$ denoting open or closed social resources) (Nou et al., 2025). Schools receive a government mental health education subsidy of αS_m , where α is the subsidy ratio and S_m is the maximum subsidy amount (Lu et al., 2021). If society opens resources (e.g., psychological counselor support), the risk cost for schools to address student mental health issues is significantly reduced, i.e., $k_{ij}^O < k_{ij}^C$.

Assumption 3: The revenue of the home-school collaboration mechanism is related to its service type. If “deep collaboration” (e.g., regular psychological counseling) is chosen, the revenue is R_{CD}^h and the collaboration cost is C_{DC}^h ; if “basic collaboration” (e.g., one-way information dissemination) is chosen, the revenue is R_{BC}^h and the cost is C_{BC}^h , where β is the subsidy intensity and S_p is the maximum subsidy amount for the collaboration mechanism.

The benefits of the home-school collaboration mechanism are directly related to the selected service type and are moderated by government financial subsidies. If “deep collaboration” is chosen (e.g., regular psychological counseling, joint assessments, and follow-up interventions), the total collaborative benefits obtained by families and schools are denoted as R_{DC}^h . The school incurs collaboration costs of C_{DC}^h but can receive a government subsidy of $\beta \cdot S_p$ to offset part of these expenses, resulting in a net cost of $C_{DC}^h - \beta \cdot S_p$ for the school.

If “basic collaboration” is chosen (e.g., one-way information dissemination, one-time lectures), the total collaborative benefits obtained by families and schools are denoted as R_{BC}^h . The school incurs collaboration costs of C_{BC}^h and typically does not receive additional subsidies.

Where: $\beta \in [0, 1]$ —the government’s subsidy intensity (proportion) for “deep collaboration” programs; S_p —the maximum subsidy amount set by the government for such programs (a fixed amount).

Assumption 4: When social professional psychological institutions (e.g., counseling centers) collaborate with schools, if the school chooses “proactive leadership,” the institution’s revenue is R_A^c ; if the school chooses “reactive response,” the institution’s revenue is R_P^c . The institution’s service cost is c_c , and it can additionally receive a government incentive of θR_C when collaborating, where θ is the incentive ratio. The institution’s willingness to collaborate depends on expected revenue and costs—the higher the incentive ratio, the stronger the collaboration tendency (Chacon et al., 2017).

Assumption 5: The total revenue of home-school-society collaboration is $R_C = R_C^h + R_C^s + R_C^c$, corresponding to the revenues of families, schools, and social institutions, respectively. The three parties distribute the revenue according to the proportion $\lambda_h : \lambda_s : \lambda_c$ (e.g., the family’s share is λ_h). A reasonable distribution proportion can enhance collaboration stability and avoid cooperation breakdown due to unequal distribution.

Assumption 6: Society supports home-school-society collaboration through the “open resources” strategy (e.g., providing special mental health funds) with a support intensity of R_s . Resource support can reduce family mental health education costs (C_f^j), school risk probabilities (k_{ij}), and enhance family participation enthusiasm through the incentive coefficient α . The greater the support intensity, the higher the psychological benefits of the collaboration system.

Assumption 7: The evolution of the system is driven by the difference in expected psychological benefits among the three parties. If the total collaborative benefit $\Delta U = U_C - U_N$ is higher than the benefit of unilateral actions, the system tends toward a cooperative equilibrium; otherwise, it degenerates into a low-collaboration state.

The evolution equations are described as: $dt/dx = f \cdot x \cdot (U_C^h - U_N^h)$, $dt/dy = g \cdot y \cdot (U_C^s - U_N^s)$, $dt/dz = h \cdot z \cdot (U_C^c - U_N^c)$, where f, g, h represent the adjustment rates of family, school, and social strategies, respectively, determined by marginal psychological benefits and cost.

TABLE 2 Payoff matrix for home-school-society tripartite game.

Family strategy	School strategy	Social strategy	Household income	School income	Social benefit
Actively participate (x)	Take the initiative to lead (y)	Open resources (z)	$R_{AO}^f - c_O^f - k_{AO}d + \alpha s_f$	$(1 - \theta)R_{AO}^s - C_{AO}^s + \beta s_s$	$R_{AO}^c + \theta R_{AO}^s - c^c$
Actively participate (x)	Take the initiative to lead (y)	Closed resources (1-z)	$R_{AC}^f - c_C^f - k_{AC}d + \alpha s_f$	$(1 - \theta)R_{AC}^s - C_{AC}^s + \beta s_s$	0
Actively participate (x)	Passive response (1-y)	Open resources (z)	$R_{PO}^f - c_O^f - k_{PO}d + \alpha s_f$	$(1 - \theta)R_{PO}^s - C_{PO}^s + \beta s_s$	$R_{PO}^c + \theta R_{PO}^s - c^c$
Actively participate (x)	Passive response (1-y)	Closed resources (1-z)	$R_{PC}^f - c_C^f - k_{PC}d + \alpha s_f$	$(1 - \theta)R_{PC}^s - C_{PC}^s + \beta s_s$	0
Actively participate (1-x)	Take the initiative to lead (y)	Open resources (z)	R^f	$-c_{AO}^s + \beta s_s$	$-c^c$
Actively participate (1-x)	Take the initiative to lead (y)	Closed resources (1-z)	R^f	$-c_{AC}^s + \beta s_s$	0
General participation (1-x)	Passive response (1-y)	Open resources (z)	R^f	$-c_{AO}^s + \beta s_s$	$-c^c$
General participation (1-x)	Passive response (1-y)	Closed resources (1-z)	R^f	$-c_{AC}^s + \beta s_s$	0

$a^f + r_f$: Family payoff under the strategy combination (i, j), where i denotes the school strategy and j denotes the social strategy.

c_j^f : Family collaboration cost under social strategy j (j=Open/Closed Resources).

$\lambda_1 < 0, \lambda_2 < 0$: Probability of mental health risk occurrence under the strategy combination (i, j).

Similarly, $\lambda_1 < 0, \lambda_2 < 0, R_{ij}^s, c_{ij}^s$, and c^c represent the payoffs and costs for schools and social institutions, respectively.

θ : Revenue-sharing ratio between schools and social institutions.

α^f, β_s^s : Government subsidies for family and school mental health education support.

In the passive participation state, families obtain regular payoff R^f , independent of the strategies of the other two parties.

Assumption 8: The stability of the collaboration system depends on the feedback mechanisms of the three parties and policy support. When government subsidies (α, β), family incentives (s_f), and revenue distribution ($\lambda_h, \lambda_s, \lambda_c$) are reasonable, the system tends to a stable cooperative equilibrium (e.g., equilibrium point E8 with significantly reduced psychological risks); if any party's revenue is insufficient or costs are too high (e.g., excessive family psychological burden), the system may degenerate into a low-collaboration equilibrium (e.g., E1, E3). Stability can be analyzed through the Jacobian matrix to quantify the evolutionary paths of collaborative strategies.

2.2 Construction of the game model

Based on the above assumptions, this study constructs a tripartite evolutionary game model among families, schools, and society. The mixed-strategy game matrix is shown in Table 2.

2.3 Model analysis

In this section, we will conduct a detailed analysis of the strategic evolution of families, schools, and society in the collaborative education of college students' mental health using the theory of evolutionary game with replicator dynamics, based on the aforementioned assumptions. By establishing a mathematical model, our aim is to reveal the evolutionary process and stability of the behavioral strategies of each subject, thereby providing theoretical support for effectively promoting the collaborative strategies among families, schools, and society (Perc et al., 2021).

(1) Evolution of family behavioral strategies

The behavioral strategy choices of families can be defined as a selection between "active participation" and "general participation." Let the probability of families choosing the "active participation" strategy be x , and its expected payoff U_f is determined by the combination of strategies of families, schools, and society. The expected payoff when families choose "active participation" is:

$$U_f = yz(R_{AO}^f - c_O^f - k_{AO}d + \alpha s_f) + y(1-z)(R_{AC}^f - c_C^f - k_{AC}d + \alpha s_f) + (1-y)z(R_{PO}^f - c_O^f - k_{PO}d + \alpha s_f) + (1-y)(1-z)(R_{PC}^f - c_C^f - k_{PC}d + \alpha s_f)$$

Among them, R_{AC}^f , R_{PO}^f , and R_{PC}^f respectively represent the payoffs obtained by families under different strategy combinations, c_O^f and c_C^f are the collaboration costs of families, k_{AO} , k_{AC} , k_{PO} , and k_{PC} , respectively represent the probabilities of psychological risks occurring under each strategy for families, d represents the maximum possible psychological risk cost in collaboration, and αs_f is the subsidy from the government to families.

When families choose the "general participation" strategy, the expected payoff is simplified to the regular payoff R_f , that is:

$$\bar{U}_f = xU_f + (1-x)R_f$$

The replicator dynamic equation for family strategies is driven by the difference in expected payoffs and can be expressed as:

$$F(x) = \frac{dx}{dt} = x(U_f - \bar{U}_f) = x(1-x)(U_f - R_f)$$

This equation indicates that when the expected payoff brought by the "active participation" strategy of families is higher than the average payoff, the proportion of this strategy in the family population will increase; conversely, it will decrease.

(2) Evolution of school behavioral strategies

The behavioral strategy choices of schools can be divided into two types: "active leadership" and "passive response." Let the probability of schools choosing the "active leadership" strategy be z , and its expected payoff U_s is:

$$U_s = xz((1-\theta)(R_{AO}^s - c_{AO}^s + \beta s_s) + x(1-z)((1-\theta)(R_{AC}^s - c_{AC}^s + \beta s_s) + (1-x)z(-c_{AO}^s + \beta s_s) + (1-x)(1-z)(-c_{AC}^s + \beta s_s))$$

Among them, R_{AO}^s , R_{AC}^s , c_{PO}^s and R_{PC}^s respectively represent the payoffs of schools under different strategy combinations, c_{AO}^s , c_{AC}^s , c_{PO}^s , and c_{PC}^s are the collaboration costs of schools, βs_s is the subsidy from the government to schools, and θ is the profit-sharing ratio between schools and society.

The expected payoff when schools choose the "passive response" strategy is:

$$U'_s = xz((1-\theta)R_{PO}^s - c_{PO}^s + \beta s_s) + x(1-z)((1-\theta)R_{PC}^s - c_{PC}^s + \beta s_s) + (1-x)z(-c_{PO}^s + \beta s_s) + (1-x)(1-z)(-c_{PC}^s + \beta s_s)$$

The average expected payoff of schools is:

$$\bar{U}_s = yU_s + (1-y)U'_s$$

The replicator dynamic equation of schools can be written as:

$$F(y) = \frac{dy}{dt} = y(1-y)(U_s - U'_s)$$

This equation indicates that when the "active leadership" strategy of schools has a higher expected payoff relative to the "passive response" strategy, schools will tend to adopt the "active leadership" strategy.

(3) Evolution of social behavioral strategies

The behavioral strategy choices of society are "open resources" and "not open resources." Let the probability of society choosing the "open resources" strategy be y , and its expected payoff is:

$$U_c = xy(R_{AO}^c + \theta R_{AO}^s - c^c) + x(1-y)(R_{PO}^c + \theta R_{PO}^s - c^c) - (1-x)yc^c - (1-x)(1-y)c^c$$

Among them, R_{AO}^c , R_{PO}^c represent the payoffs of society under different strategy combinations, cc is the resource input cost of society, and θ is the profit-sharing ratio.

When society chooses “not open resources,” the expected payoff is zero. The replicator dynamic equation of society is:

$$F(z) = \frac{dz}{dt} = z(1-z)(U_c)$$

This equation indicates that the “open resources” strategy of society will depends relative payoff. When this strategy brings positive payoffs, society will tend to choose this strategy.

(4) System stability analysis

To determine the evolutionary stability of the system, we examine the equilibrium points of the replicator dynamic equations and their stability. By solving the replicator dynamic equations of each party, the evolutionary stable points are obtained as:

$$(x^*, y^*, z^*) = \left(\frac{cf + \alpha' - \alpha\mu}{\lambda\gamma\delta\eta - rf + (1-x^*)cf}, \frac{cs + \beta' - \beta v}{(1-\lambda)\gamma\delta\eta - rs + (1-y^*)cs}, \dots \right)$$

On this basis, we use the Jacobian matrix to analyze the stability of the equilibrium points. Specifically, if the determinant “ $\text{Det}(J) > 0$ ” and the trace “ $\text{Tr}(J) < 0$ ” of the Jacobian matrix at a certain equilibrium point, then this point is a stable point; conversely, if the determinant is negative, it is a saddle point.

By analyzing the stability conditions, the evolutionary trends of the collaboration among families, schools, and society under different policy supports and collaboration costs can be obtained. For example, when government subsidies increase and social support strengthens, the system tends to a high-collaboration state $(x^*, y^*, z^*) = (1, 1, 1)$, while in the case of excessively high collaboration costs or lack of trust, the system may stabilize in a low-collaboration state.

2.4 Tripartite evolutionary stability analysis

As the core subjects of the collaborative education system for college students' mental health, the strategic choices and behavioral interactions of families, schools, and society directly influence the evolutionary direction and collaborative effectiveness of the system. Families serve as the fundamental support units for students' psychological development, schools act as the leading implementers of educational services, and society provides external guarantee for resource integration and professional services (Bronfenbrenner, 1979). Through profit sharing, cost sharing,

and risk aversion, they form a dynamic game relationship, and the stability of their strategic adjustments directly determines whether the collaborative education system can achieve long-term equilibrium (World Health Organization, 2022). Therefore, this section will analyze the asymptotic stability conditions of their strategic evolution from the perspectives of families, schools, and society respectively, reveal the impact paths of different policy parameters and collaborative mechanisms on the overall stability of the system, and provide theoretical support for optimizing the collaborative education model (Smith, 1982).

Evolutionary stability analysis aims to reveal the strategic adjustment mechanisms and long-term equilibrium states of families, schools, and society in the collaborative education game. By constructing replicator dynamic equations and Jacobian matrices, this section systematically analyzes the stability conditions of each subject's strategies and their interactive effects from both the perspectives of single-subject dynamic evolution and tripartite collaboration (Organization for Economic Co-operation and Development, 2021).

(1) Asymptotic stability analysis of families

As the fundamental decision-making subject in the game, the strategic choices of families have an important impact on the stability of the system (Ostrom, 1990). This section will analyze the evolutionary stability of families under different scenarios and explore how family behaviors tend to stabilize under different policy and environmental conditions (Zhang et al., 2020).

2.5 Construction of dynamic equations

The strategic choices of families in the game are driven by payoff differences. Let the proportion of families choosing the “active participation” strategy be $x(t)$, and their expected payoff R_{ij}^f can be expressed as:

$$R_{ij}^f = yz(\alpha\mu + \lambda\gamma\delta\eta - c_f - r_f) + y(1-z)(\alpha\mu - c_f) + (1-y)z(\alpha\mu - c_f) + (1-y)(1-z)(\alpha\mu - c_f)$$

Simplified to:

$$R_{ij}^f = \alpha\mu - c_f + yz\lambda\gamma\delta\eta - yzr_f$$

The expected payoff for families choosing “general participation” is a fixed value $\pi_f^P = \alpha'$. According to evolutionary game theory, its replicator dynamic equation is:

$$F(x) = \frac{dx}{dt} = x(R_{ij}^f - \pi_f^P) = x(1-x)[\alpha\mu - c_f - \alpha' + yz(\lambda\gamma\delta\eta - r_f)]$$

Among them, $\pi_f = xR_{ij}^f + (1-x)\pi_f^P$.

In the stability judgment of the evolutionary game model, the boundary equilibrium points and interior equilibrium conditions are key analysis dimensions. For the stability judgment of family

strategies, when the proportion of families choosing “active participation” $x^* = 0$ (i.e., complete “general participation”), if the eigenvalue $\gamma x < 0$, then $x^* = 0$ is an evolutionarily stable strategy (ESS), indicating that families generally tend not to actively participate in mental education and this state is stable; when $x^* = 1$ (i.e., complete “active participation”), if the eigenvalue $\gamma x < 0$, then $x^* = 1$ is an ESS, indicating that the state of general active participation by families is sustainable. If there exists an interior equilibrium x^* (0, 1), it must satisfy, and at this time, the stability needs to be further determined by the sign of the second derivative of the Jacobian matrix, reflecting the dynamic equilibrium state when family strategic choices are at an intermediate proportion. From a policy perspective, when the collaborative payoff satisfies $\lambda\gamma\delta\eta > r_f$ and the comprehensive effect of government subsidies and family payoffs satisfies $\alpha\mu > cf + \alpha'$, families are more inclined to actively participate in mental education; at the same time, increasing the intensity of social support η can reduce the family participation cost r_f , thereby enhancing the cooperation willingness of families and promoting the system to evolve toward a stable state of active participation.

(2) Asymptotic stability analysis of schools

The proportion of schools choosing the “active leadership” strategy is $y(t)$, and their expected payoff π_s^P is:

$$\pi_s^P = xz[\beta v + (1 - \lambda)\gamma\delta\eta - c_s - r_s] + x(1 - z)(\beta v - c_s) + (1 - x)z(\beta v - c_s) + (1 - x)(1 - z)(\beta v - c_s)$$

Simplified to:

$$\pi_s^P = \beta v - c_s + xz(1 - \lambda)\gamma\delta\eta - xzr_s$$

The fixed payoff for the passive response strategy is $\pi_s^R = \beta'$, and the replicator dynamic equation is:

$$F(y) = \frac{dy}{dt} = y(1 - y) \left[\beta v - c_s - \beta' + xz((1 - \lambda)\gamma\delta\eta - r_s) \right]$$

In the stability analysis of school strategies, the judgment of boundary equilibrium points is the core content. When the proportion of schools choosing “actively leading mental health education” $y^* = 0$ (i.e., complete “passive response”), its stability requires satisfying, indicating that schools generally adopt passive strategies and the state is stable; when $y^* = 1$ (i.e., complete “active leadership”), the stability condition requires satisfying, and at this time, the active strategy of schools becomes an evolutionarily stable strategy (ESS). The sensitivity of the interior equilibrium is reflected in that if there exists an intermediate proportion strategy y^* (0, 1), its existence depends on the dynamic balance between collaborative payoffs and costs, that is, the strategic choice of schools between active leadership and passive response needs to achieve a balance between payoffs and costs; otherwise, the system may achieve a balance between payoffs and costs toward the boundary (World Bank, 2023). At the policy level, it is necessary to optimize the collaborative mechanism to increase the payoffs for schools to actively lead, or reduce their costs through

subsidies, so as to promote the stability of school strategies toward active leadership.

(3) Asymptotic stability analysis of society

Construction of dynamic equations

The proportion of society choosing the “open resources” strategy is $z(t)$, and its expected payoff π_C^O is:

$$R_A^C = xy(k\eta - c_r)$$

The payoff for not opening resources is always 0, and the replicator dynamic equation is:

$$F(z) = \frac{dz}{dt} = z(1 - z)[xy(k\eta - c_r)]$$

The stability analysis of social strategies focuses on boundary equilibrium points and collaborative dependence effects. When the proportion of society choosing “opening mental resources” $z^* = 0$ (i.e., complete “not opening resources”), the stability condition requires satisfying, and at this time, the state of social resource closure is sustainable; when $z^* = 1$ (i.e., complete “opening resources”), the stability condition also depends on, indicating that the strategy of social resource opening becomes a stable equilibrium. The uniqueness of social strategies lies in that their stability highly depends on the cooperation levels of families (x) and schools (y): when the collaborative effect coefficient $k\eta$ is greater than the cost c_r of social resource opening, and both families and schools tend to cooperate actively ($x, y \rightarrow 1$), society is more inclined to open resources, forming a positive cycle of tripartite collaboration. Policy interventions can reduce the cost c_r of social resource opening through cost subsidy mechanisms (such as government purchasing services or tax relief), and at the same time, enhance the enthusiasm of social participation by increasing the collaborative effect coefficient $k\eta$ (such as strengthening the publicity of home-school cooperation and establishing resource sharing platforms), promoting the system to evolve toward a high-collaboration equilibrium (Seligman et al., 2009).

3 Analysis of the overall equilibrium strategy

Based on the asymptotic stability analysis of the strategic evolution of families, schools, and society, this chapter reveals the local equilibrium conditions and policy sensitivities of each subject's behavioral adjustments. However, the stability of a single subject does not equate to the maximization of the overall collaborative effectiveness of the system (Sandholm, 2010). As a dynamically complex system, the equilibrium state of collaborative mental health education for college students depends not only on the strategic choices of the three parties but also on the interactive feedback among the subjects and the synergistic effects of external policy interventions (Fehr and Fischbacher, 2003). To further explore the global evolutionary path and optimization mechanism of tripartite collaboration, this section will, from the perspective of the overall system, combine the eigenvalue analysis of the Jacobian

matrix and the simulation of government regulation parameters to deeply discuss the stability conditions of different equilibrium points and the dynamic impacts of policy levers on the effectiveness of collaborative education.

In the context of collaborative mental health education for college students, the strategic choices of families, schools, and society interact with each other, potentially leading to various equilibrium states. This section will explore the stability conditions and practical implications of different equilibrium points by analyzing the evolutionary stable strategies (ESS) of the system and the eigenvalues of the Jacobian matrix.

3.1 Analysis of evolutionarily stable strategies

3.1.1 Equilibrium point solution

Based on the tripartite replicator dynamic equations, the equilibrium points of the system can be obtained by solving the following system of equations:

$$\begin{cases} F(x) = x(1-x)[\alpha\mu - c_f - \alpha' + yz(\lambda\gamma\delta\eta - r_f)] = 0 \\ F(y) = y(1-y)[\beta\nu - c_s - \beta' + xz((1-\lambda)\gamma\delta\eta - r_s)] = 0 \\ F(z) = z(1-z)[xy(k\eta - c_r)] = 0 \end{cases}$$

By solving these equations, the following equilibrium points can be derived (Table 3).

Scenario 1: Households choose general participation ($x = 0$), schools choose passive response ($y = 0$), and society chooses not to open resources ($z = 0$). The stability condition is that the cost of household participation exceeds the benefits ($R^f + \alpha s_f < C_{DC}^h$) and the cost of school leadership exceeds the benefits ($R_{BC}^h + \beta S_p < C_{BC}^h$). In reality, this manifests as low collaboration among the government, households, and schools, resulting in inefficient mental health services, insufficient resource integration, and limited intervention effectiveness. The root causes include inadequate government financial subsidies or lack of policy guidance to motivate collaboration, as well as high collaboration costs due to cumbersome cross-departmental communication, information-sharing barriers, and ambiguous responsibilities. To address this, a dual-policy intervention system is needed: enhancing participation incentives through increased financial subsidies, voucher systems, and reward funds, while reducing collaboration costs via digital platforms, standardized protocols, clear responsibility lists, and third-party evaluations to drive a shift toward efficient collaboration.

Scenario 2: Households choose active participation ($x = 1$), schools choose passive response ($y = 0$), and society chooses not to open resources ($z = 0$). The stability condition is that household participation benefits exceed costs ($R_{DC}^h + \alpha s_f > C_{DC}^h$), but school leadership costs exceed benefits ($R_{BC}^h + \beta S_p < C_{BC}^h$) and society's cost of opening resources exceeds benefits ($R_c^h > R_c^e - c_c$). In reality, households bear excessive responsibility for psychological education, while schools lack motivation due to absent evaluation mechanisms or insufficient resources, and societal institutions remain closed due to no incentives. Policies should integrate mental health education into school evaluations, mandate home-school

collaboration mechanisms, and use government-purchased services to guide societal institutions to open resources, lowering barriers for households to access professional support.

Scenario 3: Households choose general participation ($x = 0$), schools choose active leadership ($y = 1$), and society chooses not to open resources ($z = 0$). The stability condition is that school leadership benefits exceed costs ($R_{DC}^h + \beta S_p > C_{DC}^h$), but household participation costs exceed benefits ($R_{BC}^h + \alpha s_f < C_{BC}^h$) and society's cost of opening resources exceeds benefits ($R_c^h > R_c^e - c_c$). In reality, the implementation of positive education means transforming schools into educational places with a positive atmosphere (Kristjánsson, 2012). Schools provide mental health services alone, but low household engagement and closed societal resources limit services to campuses, failing to address deeper family influences. Policies should reduce household participation costs through community outreach, parent training, and subsidies, while establishing school-society cooperation funds and encouraging professional institutions to enter schools to bridge service gaps.

Scenario 4: Households choose general participation ($x = 0$), schools choose passive response ($y = 0$), and society chooses to open resources ($z = 1$). The stability condition is that societal benefits of opening resources exceed costs ($R_c^e - c_c > R_c^h$), but household participation costs exceed benefits ($R_{BC}^h + \alpha s_f < C_{BC}^h$) and school leadership costs exceed benefits ($R_{BC}^h + \beta S_p < C_{BC}^h$). In reality, societal institutions open resources, but low participation from households and schools leads to underutilization due to lack of awareness or access. Policies should promote home-school awareness of resources, establish mechanisms for "societal resources in schools," and develop digital platforms to improve resource matching.

Scenario 5: Households choose active participation ($x = 1$), schools choose active leadership ($y = 1$), and society chooses not to open resources ($z = 0$). The stability condition is that home-school collaboration benefits exceed costs ($R_{DC}^h + R_{BC}^h + \alpha s_f + \beta S_p > C_{DC}^h + C_{BC}^h$), but societal costs of opening resources exceed benefits ($R_c^h > R_c^e - c_c$). In reality, home-school collaboration forms but lacks professional societal support, limiting services to prevention. Societal institutions avoid participation due to high costs and low returns. Universities should establish a supportive ecosystem for college students from the institutional level to promote their psychological development (Brendtro, 2006). Policies should offer tax breaks or subsidies to societal institutions participating in collaboration, establish standards, and use government endorsement to build trust.

Scenario 6: Family chooses active participation ($x = 1$), school chooses passive response ($y = 0$), society chooses open resources ($z = 1$). Stability conditions: family-society collaboration revenue exceeds costs ($R_{DC}^h + R_c^e + \alpha s_f > C_D^h C + 2c_c$). School-led costs exceed revenue ($R_{BC}^h + \beta S_p < C_{BC}^h$). Real-world performance: preliminary cooperation between families and social institutions, but schools rejects resource intervention due to management restrictions or lack of motivation, leading to fragmentation between in-school data and social services. Policy recommendations: legislate to clarify schools' obligations to open resources and incorporate them into evaluations; develop a home-school-community psychological data sharing platform to achieve cross-scenario data synchronization (Em et al., 2025).

TABLE 3 Calculation and analysis of eigenvalues of the Jacobian matrix at each equilibrium point.

Equilibrium point	Eigenvalue expression	Eigenvalue symbol conditions and analysis
$E_1 = (0, 0, 0)$	$\lambda_1 = a\mu - c_f - a'$ $\lambda_2 = \beta v - c_s - \beta$ $\lambda_3 = 0$	- If $\lambda_1 < 0$ and $\lambda_2 < 0$: Low-collaboration stability. - $\lambda_3 = 0$ indicates neutral stability (requires higher-order analysis).
$E_2 = (1, 0, 0)$	$\lambda_1 = -(a\mu - c_f - a')$ $\lambda_2 = \beta v - c_s - \beta'$ $\lambda_3 = 0$	- If $\lambda_1 < 0$ and $\lambda_2 < 0$: Stable unilateral collaboration by families. - Otherwise, it is a saddle point.
$E_3 = (0, 1, 0)$	$\lambda_1 = -a\mu - c_f - a'$ $\lambda_2 = -(\beta v - c_s - \beta')$ $\lambda_3 = 0$	- If $\lambda_1 < 0$ and $\lambda_2 < 0$: Stable unilateral collaboration by schools. - Otherwise, it is a saddle point.
$E_4 = (0, 0, 1)$	$\lambda_1 = -a\mu - c_f - a'$ $\lambda_2 = -\beta v - c_s - \beta'$ $\lambda_3 = 0$	- Society opens resources, but families and schools do not collaborate. Stability depends on λ_1, λ_2 .
$E_5 = (1, 1, 0)$	$\lambda_1 = -(a\mu - c_f - a')$ $\lambda_2 = -(\beta v - c_s - \beta')$ $\lambda_3 = k\eta - c_r$	- If $\lambda_3 > 0$: Unstable (social resources not opened). - If $\lambda_3 < 0$: Stable family-school collaboration.
$E_6 = (1, 0, 1)$	$\lambda_1 = -(a\mu - c_f - a' + \lambda\gamma\delta\eta - r_f)$ $\lambda_2 = \beta v - c_s - \beta'$ $\lambda_3 = 0$	- Collaboration between family and society without school participation. Stability is complex.
$E_7 = (0, 1, 1)$	$\lambda_1 = -a\mu - c_f - a'$ $\lambda_2 = -(\beta v - c_s - \beta' + (1 - \lambda)\gamma\delta\eta - r_s)$ $\lambda_3 = 0$	- Collaboration between school and society without family participation. Stability depends on λ_1 .
$E_8 = (1, 1, 1)$	$\lambda_1 = -(a\mu - c_f - a' + \lambda\gamma\delta\eta - r_f)$ $\lambda_2 = -(\beta v - c_s - \beta' + (1 - \lambda)\gamma\delta\eta - r_s)$ $\lambda_3 = k\eta - c_r$	- Conditions for high-collaboration stability: All eigenvalues are negative, i.e., 1. For families: $a\mu + \lambda\gamma\delta\eta > c_f +$ $c_j +$ 2. For schools: $c_s + \beta' + r_s$ 3. For society: $a' + r_f;$

Scenario 7: Family chooses general participation ($x = 0$), school chooses active leadership ($y = 1$), society chooses open resources ($z = 1$). Stability conditions: school-society collaboration revenue exceeds costs ($R_{BC}^h + R_{ce} + \beta S_p > C_{BC}^h + 2c_c$). Family participation costs exceed revenue ($R_{DC}^h + \alpha s_f < C_{DC}^h$). Real-world performance: schools collaborate with social institutions to provide services and during the growth process of college students, schools form a cooperative relationship with them, guiding college students to become people with love, responsibility, and initiative (Waters et al., 2017) but low family participation hinders the continuation of intervention effects into home scenarios. Policy recommendations: develop “family-friendly” service products to lower participation thresholds; offer point rewards and social resource redemption for active families to enhance initiative.

Scenario 8: Family chooses active participation ($x = 1$), school chooses active leadership ($y = 1$), society chooses open resources ($z = 1$). Stability conditions: tripartite collaboration revenue exceeds costs for all parties ($R_{DC}^h + R_{BC}^h + R_{ce} + \alpha s_f +$

TABLE 4 Analysis of home-school-society collaboration equilibria and policy recommendations.

Equilibrium point	Collaboration scenario	Key issues	Policy directions
E_1	Low collaboration among the three parties	Low service efficiency	Increase subsidies and reduce costs
E_2	Unilateral family collaboration	The family burden is too heavy	Force or motivate schools to participate
E_5	Home-school collaboration, with no participation from society	Lack of professional support	Promote the opening up of social resources
E_8	High collaboration among the three parties	Maximize benefits	Optimize the distribution ratio and establish a long-term mechanism

$\beta S_p > C_{DC}^h + C_{BC}^h + c_c$). Reasonable revenue distribution ratio: fairness in $\lambda h : \lambda s : \lambda c$. Real-world performance: deep tripartite collaboration, where families cooperate with schools and society provides professional support; services cover the entire chain and achieve systemic effects. Policy recommendations: establish special funds and clarify revenue distribution to ensure long-term cooperation; develop monitoring platforms and dynamically adjust policies based on psychological improvement data to enhance synergistic efficiency (Table 4).

Analysis of each equilibrium point provides targeted recommendations for policymakers to guide the system toward the high-collaboration equilibrium (E_8).

(1) Interior equilibrium analysis

In the evolutionary game model of home-school-society collaboration for college students’ mental health, the analysis of interior equilibrium points focuses on the dynamic balance where the strategic proportions of all three parties are in intermediate states. If an interior equilibrium $(x^*, y^*, z^*)_{\text{upeesilon}(0,1)3}$ exists, it must satisfy with the core being the dynamic matching of benefits and costs across all three parties’ strategic choices. Given $x^*, y^*_{\text{upeesilon}(0,1)}$, the existence of an interior equilibrium particularly depends on the balance between the costs and benefits of social resource opening—when the expected benefits of social resource opening (e.g., long-term social benefits from collaborative effects) equal its costs (e.g., service inputs from professional institutions), social strategic choices reach a critical point, thereby influencing the strategic equilibrium of families and schools.

This analysis indicates that the tripartite collaboration system may exhibit intermediate states of incomplete collaboration, whose stability depends on the effective allocation of social resources and dynamic adjustments to cost-benefit ratios. This provides a theoretical basis for “fine-tuning” policy interventions. For example, moderation subsidies or optimized benefit distribution can nudge the cost-benefit balance of social resource opening toward the equilibrium point, preventing the system from settling into extreme states of low or high collaboration.

(2) Practical implications of equilibrium points

TABLE 5 Eigenvalue sign conditions and stability conclusions for each equilibrium point.

Equilibrium point	Eigenvalue sign condition	Stability conclusion
$E_1 = (0, 0, 0)$	$\lambda_1 < 0, \lambda_2 < 0$	Stability: Low collaborative balance among the three parties (low efficiency of mental health services).
$E_2 = (1, 0, 0)$	$\lambda_1 < 0, \lambda_2 < 0$	Stability: Only families actively participate, while schools and society do not collaborate (partially effective).
$E_3 = (0, 1, 0)$	$\lambda_1 < 0, \lambda_2 < 0$	Stability: Only the school takes the initiative to lead, while families and society do not collaborate (due to limited resources).
$E_4 = (0, 0, 1)$	$\lambda_1 < 0, \lambda_2 < 0$	Stability: Only society opens up resources, while families and schools do not collaborate (resource waste).
$E_5 = (1, 1, 0)$	$\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$	Stability: Families and schools collaborate, but society does not open up resources (the effect may be limited due to insufficient resources).
$E_6 = (1, 0, 1)$	$\lambda_1 < 0, \lambda_2 < 0$	Stable conditions: Family and society collaborate, with schools not involved (further analysis of income distribution is needed).
$E_7 = (0, 1, 1)$	$\lambda_1 < 0, \lambda_2 < 0$	Stable conditions: Schools and society collaborate, and families do not participate (family costs need to be reduced).
$E_8 = (1, 1, 1)$	$\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$	Stability: High collaborative balance among the three parties (maximizing the benefits of mental health services).

In the tripartite evolutionary game model of collaborative psychological education for college students' mental health among families, schools, and society, different equilibrium points characterize differentiated collaboration states and their real-world implications:

Low-collaboration equilibrium [e.g., $E_1 = (0,0,0)$] manifests as non-cooperation among all three parties, resulting in inefficient mental health services. This reflects the typical dilemma of insufficient incentives and resource integration in current practice. Partial-collaboration equilibria demonstrate unilateral or bilateral cooperation limitations: $E_2 = (1, 0, 0)$: households alone participate actively. $E_3 = (0, 1, 0)$: schools alone take leadership. $E_4 = (0, 0, 1)$: society alone opens resources. $E_5 = (1, 1, 0)$: household-school collaboration without societal resources. These states exhibit inefficiencies such as resource wastage or intervention discontinuities due to missing stakeholder engagement. High-collaboration equilibrium [$E_8 = (1, 1, 1)$] achieves tripartite synergy, maximizing mental health service effectiveness as the system's ideal state (Table 5).

3.1.2 Analysis of stability

Equilibrium points such as E_1, E_2, E_3, E_4 , and E_5 remain stable under specific conditions. For example, E_1 maintains low collaboration because the benefits for all three parties are lower than the costs, while E_5 fails to attract social participation due

to the high costs of opening resources. $E_6 = (1, 0, 1)$ and $E_7 = (0, 1, 1)$ are conditionally stable, depending on optimized benefit distribution and reduced family costs, respectively. The stability of E_8 requires that the collaborative benefits for families, schools, and society all exceed their respective costs, i.e., the benefits of family participation, school leadership, and social resource opening must each be greater than their costs.

3.1.3 Policy interventions

Policy interventions should focus on three work together aspects: enhancing collaborative benefits, reducing collaboration costs, and promoting social participation. In particular, the following policy measures can be considered:

Optimizing benefits: Ensure positive net benefits through government subsidies (α for families, β for schools) and benefit distribution mechanisms (λ). Reducing costs: Establish special funds to lower family psychological education costs (e.g., free courses) and school service costs (e.g., facility sharing). Incentivizing society: Use tax incentives to reduce the cost of social resource opening (cc) and attract professional institutions. Example: When family subsidies ($\alpha > 0.35$), school subsidies ($\beta > 0.4$), and social incentives ($\theta > 0.25$) exceed critical values, and the benefit distribution ratio is reasonable (e.g., $\lambda = 0.4, \lambda = 0.4, \lambda = 0.2$), the system can stably converge to E_8 . Conversely, if any party's cost-benefit balance is disrupted (e.g., excessively high family participation costs), corresponding parameters (e.g., increasing α) should be dynamically adjusted to restore equilibrium.

3.1.4 Conclusion

Achieving high-collaboration equilibrium requires systematic policy support. Governments should use levers such as subsidies, resource allocation, and incentives to break the low-collaboration deadlock and establish dynamic adjustment mechanisms to respond to real-world feedback. This will promote long-term, stable, and efficient collaboration among families, schools, and society, providing sustainable institutional guarantee for college students' mental health services.

3.2 Tripartite collaborative evolution analysis considering government regulation

In the previous section, we analyzed the equilibrium points and stability conditions of the tripartite evolutionary game, revealing the strategic interactions among families, schools, and society in the collaborative mental health education of college students. However, achieving a stable evolution toward high-collaboration equilibrium often requires overcoming critical cost-benefit thresholds that spontaneous interactions among the three parties may not suffice to address. Therefore, the role of the government as an external regulator becomes particularly crucial. This section further explores how the government can intervene in the strategic choices of the three parties through policy tools (e.g., subsidies, incentives, and resource allocation) to optimize the system's evolutionary path and promote a more efficient and stable equilibrium in the collaborative education system.

As a key external regulator in the collaborative mental health education system for college students, the government directly influences the strategic choices of families, schools, and society through policy tools such as subsidies, legislation, and resource allocation. This section incorporates government regulation parameters (α , β , θ , S_m , S_p , etc.) into the evolutionary game model to analyze their intervention paths and effects on the system equilibrium.

Stability changes of equilibrium points under government regulation taking the high-collaboration equilibrium $E8 = (1, 1, 1)$ as an example, its stability conditions require that the collaborative benefits of families, schools, and society all exceed their respective costs. The specific core logic is as follows:

When the government's subsidy ratio for families (such as α) is sufficient to make the net benefit of families' "active participation" exceed that of "general participation" (i.e., the benefit of families' participation in psychological education R_{DC}^h minus the cost C_f is still higher than the benefit of passive participation R_f), the family strategy will stabilize at "active participation." Here, α is the subsidy ratio, and S_f is the maximum subsidy amount for families, indicating that the subsidy must reach a critical value to cover the participation costs of families and provide additional incentives.

The stability of the school's "active leadership" strategy requires that its benefits (such as the sum of the government subsidy βS_s and the mental health service benefit) are higher than the difference between the benefit of "passive response" (R_{BC}^h) and the collaboration cost (C_s). Here, β is the school subsidy ratio, and S_s is the maximum subsidy amount for schools, indicating that the government needs to reduce the net cost of schools' active participation through financial support.

The stability condition for society's "open resources" strategy is that its benefits (such as the cooperation benefit R_c^e and the government incentive θR_c) are higher than the difference between the benefit of "not opening resources" (R_c^h) and the opening cost (C_c). Here, θ is the social incentive ratio, and R_c is the total social cooperation benefit, indicating that the resource opening costs of social institutions need to be compensated through incentive mechanisms.

$$\begin{cases} \text{Family: } \alpha\mu + \lambda h\gamma\delta\eta > cf(1 - \tau) + \alpha' + rf \\ \text{University: } \beta v + \lambda_s\gamma\delta\eta > c_s(1 - \tau) + \beta' + r_s \\ \text{Society: } k\eta(1 + \theta) > cr(1 - \tau) \end{cases}$$

The subsidy ratios (α , β) and the incentive ratio (θ) in the above conditions need to reach specific critical values (such as $\alpha > 0.35$, $\beta > 0.4$, $\theta > 0.25$) to promote the three parties' strategies to stabilize in a high-collaboration state. For example, when the family subsidy α exceeds the critical value, the family participation rate will significantly increase and remain stable, and the same applies to the regulatory parameters of schools and society. These critical values provide a quantitative basis for the government to formulate policies, ensuring that the subsidy and incentive efforts are sufficient to break through the "non-collaborative inertia" of the three parties and form a stable collaborative education mechanism. The specific mechanisms through which these key government parameters influence the strategic choices of families, schools, and society are systematically summarized in [Table 6](#).

3.3 Simulation of government regulation scenarios

Scenario 1: In the single-policy scenario where only the government's subsidy ratio α for households is adjusted, the proportion x of households choosing "active participation in psychological education" significantly increases, reflecting the direct incentive effect of subsidies on households' willingness to participate. However, since the school subsidy β and social incentive θ are not adjusted simultaneously, schools may maintain a "passive response" strategy due to higher dominant costs than benefits, and social institutions may remain "unwilling to open resources" due to the lack of cooperation incentives. This leads the system to converge to a partial collaboration equilibrium point $E5 = (1, 1, 0)$, where households participate actively, schools take the initiative, but society does not open resources. The limitation of this scenario is that single policies struggle to drive tripartite coordination, and potential strategy lags in schools and society may cause resource misallocation. For example, household needs and school services lack support from professional social resources, affecting the overall effectiveness of mental health services.

Scenario 2: When the government implements a coordinated regulation policy by simultaneously increasing the household subsidy α , school subsidy β , and social incentive θ , the collaboration motivation of the three parties is fully activated: households reduce participation costs due to increased subsidies, schools enhance their willingness to take the initiative due to financial support, and social institutions are more inclined to open resources due to increased incentives. This systematic policy intervention can rapidly drive the system to converge to a high-collaboration equilibrium point $E8 = (1, 1, 1)$, i.e., the ideal state where households participate actively, schools take the initiative, and society opens resources. Data examples show that when $\alpha = 0.6$, $\beta = 0.5$, and $\theta = 0.4$, the tripartite collaboration probability can increase to 85%, indicating that coordinated regulation effectively breaks the low-collaboration deadlock, forms a positive cycle of tripartite linkage through multiple incentives, and significantly improves the efficiency and effectiveness of collaborative mental health education.

Scenario 3: Dynamically adjusting the profit distribution ratios (λ_h , λ_s , λ_n) for households, schools, and society can effectively prevent any party from withdrawing from collaboration due to insufficient profits, thereby enhancing system stability. For instance, when setting $\lambda_h = 0.4$, $\lambda_s = 0.4$, and $\lambda_n = 0.2$, the profit distribution among the three parties becomes more balanced: households and schools, as core education entities, receive higher profit shares to incentivize their continuous collaborative investment; although social institutions have a slightly lower share, their willingness to open resources is maintained through stable profit expectations. This distribution model alleviates the risk of cooperation breakdown caused by interest imbalance, particularly in the early stages of collaboration or scenarios with significant resource input disparities. Through an institutionalized profit distribution mechanism, it guides the three parties to form long-term stable collaborative relationships and promotes the system's evolution toward high-collaboration equilibrium.

TABLE 6 Correlation mechanisms between government regulation parameters and tripartite strategies.

Policy tools	Regulated object	Action path	Mathematical representation
Family subsidy	Family participation enthusiasm	Reduce the cost of psychological education and increase the benefits of “active participation.”	αs_f
Family subsidy	School-led willingness	Cover collaboration costs (such as training expenses) and incentivize the “proactive” strategy.	βs_s
Social motivation	Degree of resource openness	Compensate for the cooperation costs of social institutions through tax incentives or special funds.	θRc
Income distribution adjustment	Stability of tripartite collaboration	Adjust the proportion of income from families, schools and society ($\lambda_h, \lambda_s, \lambda_c$).	$\lambda_h : \lambda_s : \lambda_c$
Risk sharing mechanism	Psychological risk cost	The government assumes some risks (such as establishing a psychological crisis intervention fund) to reduce $k_{ij}d$.	$k_{ij} \rightarrow k_{ij}(1 - \tau)$

TABLE 7 Comparison of equilibria before and after government coordination.

Regulation strategy	Household participation rate x	School initiative rate y	Social openness rate z	Convergence speed
No regulation (Baseline)	0.35	0.28	0.12	Slow
Single subsidy ($\alpha = 0.5$)	0.72	0.31	0.15	Medium
Coordinated regulation ($\alpha = 0.5, \beta = 0.4, \theta = 0.3$)	0.89	0.85	0.78	Fast

3.4 Numerical simulation validation

In the evolutionary game model of home-school-society tripartite collaboration for college students’ mental health, government regulation strategies significantly influence the strategic choices of the three parties and the system’s equilibrium state. Numerical simulations comparing the equilibrium results under three scenarios—no regulation, single subsidy, and coordinated regulation (see Table 7)—clearly reveal the practical effects of different policy interventions.

In the unregulated (baseline) state, the household participation rate ($x = 0.35$), school initiative rate ($y = 0.28$), and social openness rate ($z = 0.12$) all remain low, with slow system convergence. This indicates that without external intervention, the three parties generally prefer low-collaboration strategies due to high collaboration costs and insufficient benefits, leading to a “fragmented” dilemma in mental health services.

The single-subsidy policy (adjusting only $\alpha = 0.5$) significantly increases the household participation rate ($x = 0.72$), but the improvements in the school initiative rate ($y = 0.31$) and social openness rate ($z = 0.15$) are limited, with a moderate convergence speed. This confirms the limitation of single policies: while household enthusiasm is stimulated, schools maintain passive responses due to the lack of financial support (β unadjusted), and social institutions remain resource-closed due to the absence of incentives (θ unadjusted), resulting in the system only achieving a partial equilibrium of “active household participation, low collaboration from schools and society” (e.g., E5). The overall collaborative efficiency is insufficient.

Coordinated regulation ($\alpha = 0.5, \beta = 0.4, \theta = 0.3$) demonstrates the advantages of systematic policies: the household participation rate ($x = 0.89$), school initiative rate ($y = 0.85$), and social openness rate ($z = 0.78$) all significantly increase, with the fastest system convergence speed. This shows that simultaneously

adjusting subsidy ratios (α, β) and incentive coefficients (θ) can form a tripartite linkage effect—households reduce participation costs due to subsidies, schools enhance initiative due to financial support, and social institutions open resources due to profit incentives—ultimately driving the system rapidly toward high-collaboration equilibrium (E8) (see Table 7 for details).

3.5 Analysis of tripartite coevolution considering government regulation

In the three party evolutionary game model of college students’ mental health cooperative education, the quantitative influence of government regulation parameters such as subsidy ratio α , β , θ on the choice of three party strategies can be deeply revealed through numerical examples and sensitivity analysis. Taking the action mechanism of family subsidy ratio α on family participation rate x as an example, it has influence through cost-benefit reconstruction mechanism. The increase of α will directly increase the expected benefit of family “active participation” strategy and reduce the net cost. Widening the difference in returns from “average participation” creates positive incentives that drive up the household participation rate x . For quantitative analysis, the initial parameters are set as family participation cost $c_f = 0.5$, other subsidy related parameters $\alpha' = 0.2$, synergy benefit coefficient $\mu = 1.0$, comprehensive benefit term $-c_{AO}^s + \beta s_s$, family risk cost, and the school dominance rate/and social openness rate/are fixed to simulate the middle cooperation level scenario. When $\alpha \leq 0.35$, the cost of family active participation is higher than the benefit, and the family tends to participate generally. For example, when $\alpha = 0.2$, the benefit difference is -0.1 , the steady participation rate is 0, and the simulation result $x = 0.05$; When $\alpha > 0.35$, the profit difference is positive, and the participation rate increases

significantly. For example, when $\alpha = 0.4$, the profit difference is 0.12, the steady participation rate is 0.63, the simulation result $x = 0.65$, when $\alpha = 0.6$, the profit difference is 0.34, the steady participation rate reaches 1, and the simulation result $x = 0.98$ is close to the complete participation state, which reflects the threshold effect and the marginal increasing law. Moreover, the analytical solution is highly consistent with the simulation results to verify the reliability of the model. This indicates that the government needs to set α above 0.35 to break through the cost-benefit threshold, simultaneously optimize β and θ to avoid single policy leading to families fighting alone, and enhance policy effectiveness by reducing the inherent cost of families or improving the synergistic benefit coefficient, which provides quantitative basis for formulating differentiated subsidy policies and emphasizes the necessity of multi-agent collaborative intervention (see Table 8 for details).

(1) The influence of β on y

In the home-school-community three-way evolutionary game model of college students' mental health cooperative education, the influence of government regulation parameters on three-way strategy can be further clarified through numerical examples and sensitivity analysis. Taking the effect mechanism of β on y , β increases the net benefit by covering the cost of school collaboration (such as teacher training cost). The initial parameters are set as school collaboration cost $c_s = 0.6$, other subsidy related parameters $\beta' = 0.3$, synergy benefit coefficient $\nu = 1.2$, comprehensive benefit term $(1 - \lambda)\gamma\delta\eta = 0.7$, school risk cost $r_s = 0.4$, and fixed family participation rate/and social openness rate $z = 0.6$. The results show that when $\beta = 0.3$, the benefit difference is -0.15 , Schools maintain passive coping (steady state dominance 0, simulation $y = 0.08$), while the return difference is 0.22 when $\beta = 0.5$, steady state dominance 0.75 (simulation $y = 0.72$), return difference 0.49 when $\beta = 0.7$, schools are completely active dominant (steady state $y = 1$, simulation $y = 0.95$), and y increases about 64% when β increases from 0.3 to 0.5, but the marginal effect decreases when $\beta > 0.7$, which needs to be combined with financial budget optimization (see Table 9 for details).

(2) Influence of social incentive ratio θ on social openness ratio z

The influence of social incentive ratio θ on social openness rate z is reflected in compensating the cost of resource opening θR_c^c by revenue sharing c_c , encouraging social institutions to choose the strategy of "open resources," setting initial parameters as social opening cost $c_c = 0.8$, synergy coefficient $k\eta = 1.0$, social revenue $R_{AO}^c = 0.5$, $R_{PO}^c = 0.3$, under different school strategies, and fixing $x = 0.8$, $y = 0.7$. The data show that when $\theta = 0.1$, social net revenue -0.25 , maintaining resource closure (steady state $z = 0$, simulation $z = 0.03$), net return 0.18 when $\theta = 0.3$, steady state open rate 0.82 (simulation $z = 0.79$), net return 0.45 when $\theta = 0.5$, complete open (steady state $z = 1$, simulation $z = 0.97$), and z rises rapidly when $\theta > 0.25$. The government can optimize resource investment through stepped incentives (such as $\theta = 0.4$) (see Table 10 for details).

(3) Simulation of comprehensive regulation effect

The simulation of comprehensive regulation effect shows that the convergence steps of the system exceed 100 and the cooperation

TABLE 8 Difference of family income and participation rate under different family subsidy ratio α .

α value	Family benefit difference ($U_f - R_f$)	Steady-state participation rate (analytical solution)	Simulation result x ($t = 10$)
0.2	-0.1 (Cost > Benefit)	0.0 (No participation)	0.05
0.4	0.12 (Benefit > Cost)	0.63	0.65
0.6	0.34	1.0 (Full participation)	0.98

TABLE 9 Difference and dominance ratio of school income under different school subsidy ratio β .

β value	School benefit difference	Steady-state leadership rate	Simulation result y ($t = 10$)
0.3	-0.15	0.0	0.08
0.5	0.22	0.75	0.72
0.7	0.49	1.0	0.95

TABLE 10 Numerical analysis of the influence of social incentive ratio θ on social openness rate.

θ value	Social net benefit	Steady-state openness rate z^*	Simulation result z ($t = 10$)
0.1	-0.25	0.0	0.03
0.3	0.18	0.82	0.79
0.5	0.45	1.0	0.97

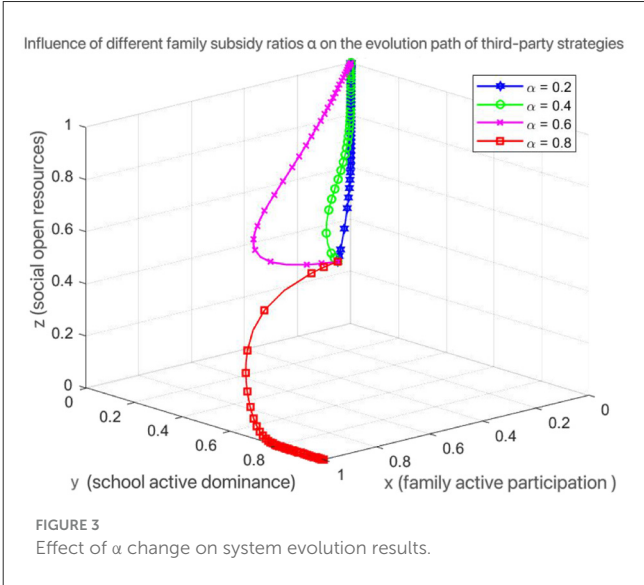
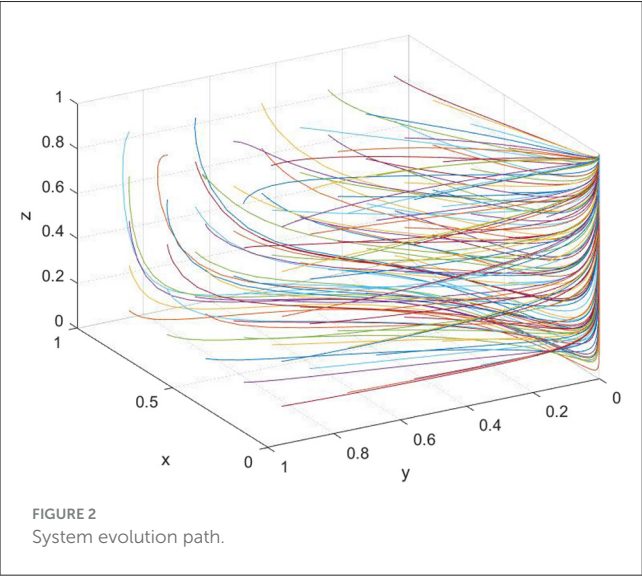
rate is low without regulation ($x = 0.35$, $y = 0.28$, $z = 0.12$), when only adjusting $\alpha = 0.5$ and $\beta = 0.4$, the home-school collaboration rate increased significantly ($x = 0.89$, $y = 0.78$), but the social openness rate is only 0.45, while the cooperative regulation $\alpha = 0.6$, $\beta = 0.5$, $\theta = 0.4$, the tripartite cooperation rate reaches 0.95, 0.92, 0.88, respectively, and the convergence steps are only 25. It is verified that the multi-dimensional policy linkage can significantly accelerate the system to converge to the high cooperative equilibrium E, and when $\alpha > 0.5$, $\beta > 0.4$, $\theta > 0.3$, the cooperation rate exceeds 85% (see Table 11 for details).

(4) Policy recommendations supplement

Based on the above analysis, policy recommendations include differentiated subsidy design, such as setting $\alpha \geq 0.5$ for low-income families, increasing β to 0.6 for rural schools, and dynamically adjusting θ for social institutions according to service effects (For example, $\theta = 0.3 + 0.1 \times \text{service satisfaction}$). At the same time, the government needs to monitor the critical values of family $\alpha > 0.35$, school $\beta > 0.4$ and society $\theta > 0.25$ in real time, and update the parameters according to the cooperation effect every semester to form a dynamic adjustment mechanism to continuously optimize the efficiency of home-school-community collaborative education.

TABLE 11 Cooperation rate and convergence steps of home-school-community under different control combinations.

Regulation combination	Family	School y	Social z	Convergence steps (Iterations)
No regulation (Baseline)	0.35	0.28	0.12	> 100
$\alpha = 0.5, \beta = 0.4$	0.89	0.78	0.45	40
$\alpha = 0.6, \beta = 0.5, \theta = 0.4$	0.95	0.92	0.88	25



4 Simulation analysis of three-way evolutionary game

In order to verify the applicability of home-school-community three-way evolutionary game model in college students' mental health collaborative education, and further analyze the influence of government regulation parameters (such as family subsidy ratio α , school subsidy ratio β , social incentive ratio θ), cooperation cost, resource opening income and other key variables on the system evolution path and results, this section carries out numerical simulation analysis based on the ideal evolutionary stable state (1, 1, 1).

Matlab R2022b was used for simulation. Referring to the replication dynamic equation and parameter setting in Section 1, the evolution path and equilibrium evolution trend of the strategy under the change of key variables were analyzed one by one on the basis that the initial probability of the strategy was 0.4.

4.1 Three-way evolutionary path simulation

Substituted the basic parameters into the model, and set the initial strategy selection probability as: family active participation $x = 0.4$, school active dominance $y = 0.4$, social open resources $z = 0.4$. The simulation evolution path is shown in Figure 2.

As can be seen from Figure 2, the final evolution of the system tends to a stable state (1, 1, 1), indicating that families, schools, and society finally choose to cooperate in mental health services, verifying the feasibility and stability of the evolutionary game

model under the background of collaborative education. Next, this paper will examine the impact of key policy parameters on evolutionary paths.

4.2 Influence of family subsidy ratio α on evolution results

In order to explore the impact of government subsidies on the evolution of the system, let α be 0.2, 0.4, 0.6, 0.8, and the other parameters remain unchanged. The simulation results are shown in Figure 3.

Figure 3 shows that as α increases, the proportion of families choosing "active participation" increases significantly. When $\alpha < 0.3$, the system tends to (0, y , z), and the family participation intention is insufficient; when $\alpha > 0.5$, the family strategy converges rapidly to 1, and the system evolves to a highly cooperative state. It shows that increasing family subsidies properly can effectively stimulate their cooperative power, which is the key to building a stable cooperative system.

4.3 The influence of β ratio of school subsidy on evolution results

In order to study the promotion effect of school subsidy on system collaboration, β is set to be 0.25, 0.4, 0.6 and 0.8. The simulation results are shown in Figure 4.

Figure 4 shows that the proportion of school “active” strategies increases significantly with β . The critical threshold of β is between 0.35 and 0.45. Below this value, schools tend to be passive; above this value, active strategies quickly dominate and the system evolves more easily into a cooperative state $(x, 1, z)$. Therefore, enhancing school policy support will enhance their dominant willingness and reduce the risk of collaboration lag.

4.4 Influence of social incentive ratio θ on evolution results

Set $\theta = 0.1, 0.3, 0.5, 0.7$, and examine their impact on social resource opening strategies. The simulation results are shown in Figure 5.

Figure 5 shows that below $\theta 0.25$, society remains essentially “closed” ($z = 0$); above $\theta 0.4$, the proportion of openness rises sharply, eventually tending to 1. It shows that the cooperation tendency of social institutions is highly sensitive to incentive

mechanism, and the design of revenue sharing system is the core of promoting resource flow.

5 Conclusion

Based on the realistic demand of college students’ mental health collaborative education, this paper systematically analyzes the key factors of participation strategies by constructing an evolutionary game model involving family, school and society, and draws the following main conclusions and practical enlightenment.

For families, their participation strategies are mainly influenced by government subsidies, participation costs and expected benefits. Model simulations show that higher government subsidies can significantly increase the probability of families choosing “active participation,” especially when combined with low participation costs and high expected returns. However, when the cost of family participation is too high or the expected benefits are insufficient, their willingness to participate will be significantly inhibited.

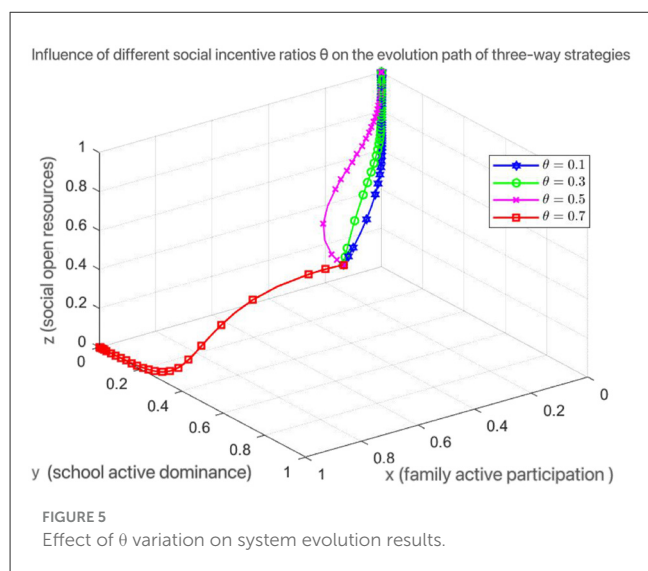
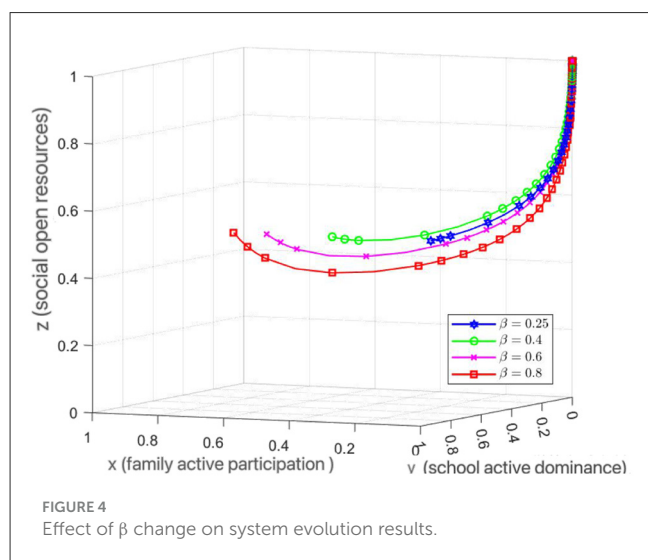
It is suggested that government departments should establish differentiated subsidy policies: raise subsidy standards for families with economic difficulties (Heng et al., 2025), and carry out universal education on mental health knowledge to reduce their psychological threshold for participation (Chin et al., 2024). Schools should optimize the home-school communication mechanism, reduce parents’ time cost through digital platform (Stephan et al., 2019), and improve the convenience of participation.

For schools, the choice of dominant strategy mainly depends on financial support, professional resources supply and risk management ability. Simulation results suggest that when government support covers at least 40% of the mental health service costs, the school’s “active leadership” strategy is more likely to reach a stable equilibrium under the model conditions. In addition, the openness of social resources directly affects school service effectiveness, and there is a significant positive correlation between the two.

The administrative department of education shall improve the special fund guarantee mechanism for mental health and establish a scientific performance appraisal system (Thon et al., 2022). At the same time, schools need to strengthen the construction of professional teams, improve risk response capabilities, and actively build cooperation platforms with social institutions to realize resource sharing and complementary advantages.

For social institutions, resource opening decisions are mainly influenced by revenue allocation ratio, operating cost, and synergy effect. When the proportion of income distribution exceeds 30%, the willingness of social institutions to choose “open resources” is significantly enhanced. It is worth noting that the participation of families and schools will amplify the marginal benefits of social resources through synergy, forming a positive feedback mechanism.

It is suggested to establish government-led income distribution guidelines and improve the psychological service procurement mechanism involving social institutions. At the same time, supporting policies such as tax preference and brand publicity can be adopted to reduce the operating costs of social institutions and enhance their enthusiasm for continuous participation.



International evidence demonstrates that the family-school-community collaborative education mechanism yields significant improvements in adolescent mental health outcomes. For instance, In Ghana, research shows that support from parents, classmates, and teachers significantly boosts student engagement (Ansong et al., 2017). In South Korea, Gyeonggi Province's "Shared School Program" has promoted cooperation among universities and community organizations to support youth education and wellbeing. In Cambodia, recent national endeavors have focused on enhancing research capacity and scholarly pursuits within higher education institutions, recognizing the foundational role of robust academic ecosystems in fostering student development, including psychosocial wellbeing (Em and Khampirat, 2025). While detailed quantitative evaluations are limited, reports indicate improvements in mental health support accessibility and service coordination. In the United States, federal initiatives such as the School-Based Mental Health Services program have allocated substantial funding—over \$500 million in recent years—to expand mental health workforce capacity in schools. These efforts have led to improvements in crisis intervention outcomes and access to care, although success rates vary across districts. These cases collectively demonstrate that tripartite collaboration not only enhances mental health service efficiency but also substantially improves student psychological wellbeing. This study establishes a theoretical foundation and practical framework for university mental health education through such collaborative mechanisms, with potential to achieve similarly positive outcomes in practice.

This study reveals the inherent mechanism of home-school-community collaboration through evolutionary game theory, quantitatively analyzes the impact of policy intervention on system equilibrium, and provides theoretical basis for improving college students' mental health service system. Future research can further explore the following directions: one is to introduce time delay effect to analyze the dynamic response process of policy implementation; the other is to investigate the impact of regional differences on synergy effects; and the third is to study the transformation effect of digital technology on traditional synergy models. These expansions will help to build a more accurate and efficient new model of collaborative education.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

JG: Data curation, Writing – review & editing, Writing – original draft, Investigation. FT: Data curation, Writing – review & editing.

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Supplementary material

The Supplementary Material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/feduc.2025.1660557/full#supplementary-material>

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