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Optimal control of *Lantana camara*: an entropy-based sustainable strategy

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Framing control policies to mitigate the impact of invasive plants on indigenous biodiversity within the Sustainable Development Goals (SDG) framework is the primary objective of this work. Using reported ecological dynamics of the invasive species *Lantana camara*, we develop a minimal three-species network model, where each node follows generalized Lotka-Volterra (GLV) dynamical equations. Employing Lie algebra and network control theory, we establish the model's controllability and accessibility criteria. Through nonlinear optimization programming, we derive sustainable policies for controlling abundances of *Lantana camara*. We have also used Shannon entropy as an indicator to assess the sustainability of these optimal policies. The analysis of the sensitivity measured using this technique reveals that the control strategy is critically dependent on the ratio of the intrinsic growth rates of the *Lantana camara* and the control plant. Thus, we get a modular algorithmic decision support mechanism for designing control policies to manage *Lantana camara* abundances.

KEYWORDS

***Lantana camara*, generalized Lotka-Volterra, Shannon entropy, sustainable development goals, sensitivity analysis**

1 Introduction

Strategic control of invasive plants in an ecosystem is a global issue, as they are one of the major causes of biodiversity loss, environmental damage, and a threat to endangered native species. In this work, we focus on controlling the invasive species *Lantana camara*, which is one of the primary drivers of imbalance between local and global plant diversity (Kohli et al., 2006; Venkataraman, 2015; Hiremath et al., 2018; Sivakumar et al., 2018; Kumar et al., 2021). This complex process of invasion leads to a state of dominance by the invasive species in the ecosystem, which we have termed as *diseased state*. We also know that *Lantana camara* promotes the growth of deleterious microbes, which decrease the population of beneficial microbes in the soil (Fan et al., 2010; Kumar et al., 2021) and implicitly hinder the growth of other plants. The main goal of this work is to propose a model and analyze it to guide the diseased ecosystem to a possible desired healthy state (Montoya et al., 2006; Jones et al., 2018; Weidlich et al., 2020; Singh et al., 2022) in a sustainable manner as suggested in the United Nations Sustainable Development Goals (UN-SDG) (United Nations General Assembly, 2015) using suitable external inputs. We have thus defined two-fold objectives: first, to manage the abundance of the *Lantana camara* population, and second, to provide a possible mathematical framework for addressing important sustainability issues related to controlling invasive plants in general. We derive our theoretical model systematically by following some of the basic ecosystem management rules.

The basic ecological observations incorporated into our model framework are as follows. For our control objectives, we introduce a native plant into the ecosystem, which helps control the abundance of *Lantana camara* and also increases soil microbe abundance. Some studies demonstrate how to utilize plant-plant interactions to regulate the abundances of plants within a spatial distance threshold from each other in the ecosystem (Sakai and Suzuki, 1954; Joshi, 1991). This interaction also has its own benefits as certain plants are known to improve soil quality by increasing and promoting the growth of good microbes in the soil (Hamilton and Frank, 2001; Wardle et al., 2004; Singh et al., 2014). It has been found in certain field studies (Joshi, 1991) that some non-invasive plants like *Ipomoea staphylina*, *Mimosa pudica* (Mimosa has also added benefits of being a leguminous plant) can act as being control plants, which further provides a practical basis to our theoretical modeling and analysis. From these basic important observations, we propose a minimal model, which consists of these two fundamental interactions i.e. plant-plant interactions and the plant-microbe interactions, and analyze this model for possible sustainable control strategies of invasive species Lantana camara. The dynamics of each nodal variable is defined by generalized Lotka-Volterra (GLV) model. The control strategies in the model are incorporated using nonlinear control theory (Liu and Barabási, 2016; Angulo et al., 2019). This is a basic minimal model specially designed to incorporate control theoretical technique and can be extended to more general interactions.

The behavior of such ecological systems can be well studied within the framework of population dynamics where the growth of the population are represented by dynamical systems. In these models, we study how the population states of species of the system change over time due to interactions within individuals of the same species and with individuals of other species present in the ecosystem. Thus we set up an optimal control problem whereby we want to find the suitable optimal control inputs that can steer the population state, evolving under the proposed GLV dynamical system, to a desired state (Kalman and Soc,1963; Brockett, 1973; Nijmeijer and van der Schaft, 1990; Conte et al., 1999; Liu et al., 2011; Brockett, 2015; Liu and Barabási, 2016; Angulo et al., 2019).

2 Model description

Components of an ecosystem such as plants, animals, soil microbes, etc. interact in a complex manner as shown in Figure 1a and can be represented by a network as shown in Figure 1b. In such networks defined by a graph $G(V,E)$, ecological agents are represented by the nodes ($\mathbf{V}$) and interactions by edges ($\mathbf{E}$) among them (Cohen, 1968; Roberts, 1978; Angulo et al., 2019). In our study, we represent the dynamics of the individual nodes by GLV model (May and Leonard, 1975). Once we construct the network dynamical system, we then ask how we can steer the system in a finite amount of time to achieve a certain desired state by suitable input signals to the chosen nodes (ecological agents) of the network as shown schematically in Figure 1b. For the analysis and solution to this situation, network theory-based controllability (Liu et al., 2011; Liu and Barabási, 2016; Angulo et al., 2019) predictions have been systematically implemented in this work.

The minimal ecological model we propose in this work is a three-node network, whose individual dynamics is governed by nonlinear GLV (Goel et al., 1971; May and Leonard, 1975; Mambuca et al., 2022), with the interacting edges built from experimental observations (Venkataraman, 2015; Sivakumar et al., 2018; Kumar et al., 2021). We define the state vector $x = [x_1, x_2, x_3]^T$ representing the abundances of *Lantana camara* (x_1), control-plants (x_2), and soil

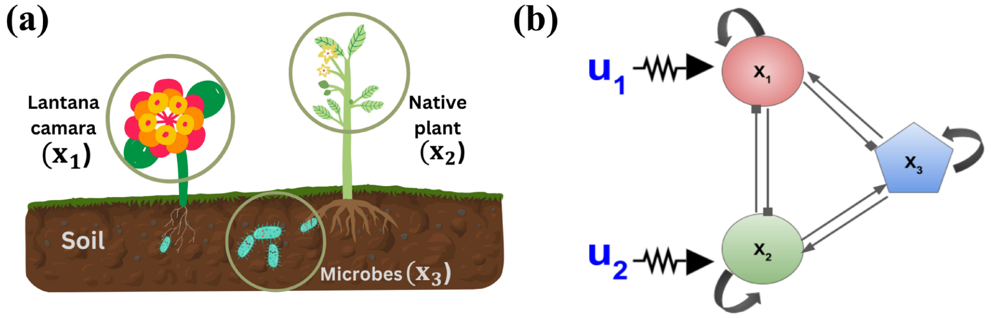

FIGURE 1
Schematic of the ecological system, its modeling. **(a)** Shows the eco-system of the three interacting species: *Lantana camara*, control plants, and microbes. **(b)** A network representation of GLV-type interactions among ecological agents. The abundances of *Lantana camara* $x_1(t)$, control plants $x_2(t)$, and microbes $x_3(t)$ as nodes in the network representing the dynamical system. Control inputs $u_1(t)$ (reducing *Lantana camara*) and $u_2(t)$ (enhancing control plants) act on x_1 and x_2 respectively, with arrows indicating the nodes at which the control is applied.

microbes (x_3). The interaction strength matrix (the adjacency matrix) $A = \left[a_{ij}\right]_{3x3}$ with $a_{ij} \in (0,1)$ quantifies the relative strength of interactions between the nodes (the participating ecological agents) in the GLV model. The uncontrolled dynamical equations are given by

$$\frac{dx}{dt} = f_0(x) = \mathrm{diag}(x)[Ax + r], \quad A = \begin{bmatrix} -a_{11} & -a_{12} & a_{13} \\ -a_{21} & -a_{22} & a_{23} \\ -a_{31} & a_{32} & -a_{33} \end{bmatrix}, \tag{1}$$

$$\mathrm{diag}(x) = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix}, \quad r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

where r_i $(i = 1, 2, 3)$ represent the intrinsic growth rates adjusted for carrying capacities of the ecological agents and where $\mathrm{diag}(x)$ denotes the diagonal matrix with diagonal entries from vector x. The state variables x_1, x_2, and x_3 represent normalized population densities (biomass fractions) and are dimensionless quantities satisfying $0 \le x_i \le 1$. The biomass proportions are normalized such that the total biomass sums to unity, with plants contributing 85% and microbes 15% to the total, ensuring the model accounts for relative biomass contributions while maintaining ecological relevance.

The time variable t is measured in months. The intrinsic growth rates r_i have units of inverse time (time^{-1}), representing per capita growth rates. The interaction coefficients a_{ij} are dimensionless parameters in the interval $(0, 1)$, quantifying the relative strength of interspecific interactions. The control inputs $u_1(t)$ and $u_2(t)$ have units of time^{-1}, representing the rate of change in population density induced by external interventions such as removal/addition of biomass, habitat modification, or resource allocation. The control vector fields f_1 and f_2 are dimensionless unit vectors indicating which state variables are directly affected by the controls. This dimensional consistency ensures that all terms in the dynamical equations have units of time^{-1}, representing rates of change of the dimensionless population densities.

In our model, we impose two control signals $u_1(t)$ and $u_2(t)$ on the abundance variables of *Lantana camara* (x_1) and control-plant (x_2), respectively. The choice of these driver nodes is based on the maximum matching algorithm applied to our system (Brockett, 1973; Angulo et al., 2019), which identifies suitable nodes at which control inputs can be effectively applied. After incorporating the control input signals into Equation 1, we have the following control equation:

$$\frac{dx}{dt} = f_0(x) + \sum_{j=1}^{2} u_j f_j(x); \quad f_1(x) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; f_2(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{2}$$

where, it was assumed that the susceptibilities (f_j) of the species to input are constant and equal to one.

3 Methods

The general controllability problem for our ecological system can be defined as steering the system to reach a desired target state x_D from a given initial state x_0 in finite time using appropriate control input signals $u_j(t)$. To determine whether such steering is possible, we must analyze the system's accessibility and controllability properties (Brockett, 1973; Sussmann, 1987; Nijmeijer and van der Schaft, 1990; Conte et al., 1999). For the control-affine system given in Equation 2, where $f_0(x)$ represents the drift dynamics and $f_j(x)$ are the control vector fields, accessibility is determined through the Lie algebra rank condition (LARC). The accessibility distribution at a point x is constructed from the vector fields and their iterated Lie brackets as shown in Equation 3:

$$\mathcal{A}(x) = \mathrm{span}\{f_1(x), f_2(x), [f_0, f_1](x), [f_0, f_2](x), [[f_0, f_1], f_0](x), \dots\}, \tag{3}$$

where the Lie bracket of two vector fields X and Y is defined as in Equation 4:

$$[X, Y](x) = \frac{\partial Y}{\partial x} X(x) - \frac{\partial X}{\partial x} Y(x). \tag{4}$$

The system is accessible at x if the accessibility distribution spans the entire state space, i.e., $\dim(\mathcal{A}(x)) = n$, where n is the dimension of the state space. This is verified by constructing the accessibility distribution matrix given in Equation 5:

$$\mathcal{C}(x) = [f_1, f_2, [f_0, f_1], [f_0, f_2], [[f_0, f_1], f_0], [[f_0, f_2], f_0], \dots], \tag{5}$$

and checking the rank condition given in Equation 6:

$$\mathrm{rank}[\mathcal{C}(x)] = n. \tag{6}$$

is crucial to distinguish between accessibility and controllability (Sussmann, 1987). Accessibility ensures that the reachable set from any initial state has a non-empty interior, meaning a neighborhood of any target state can be reached. Controllability, on the other hand, guarantees that any target state can be reached exactly in finite time. For control-affine systems with drift, accessibility is a necessary but not always sufficient condition for local controllability (Nijmeijer and van der Schaft, 1990; Conte et al., 1999). Additional conditions, such as Sussmann's condition for small-time local controllability, may need to be verified (Sussmann, 1987).

For our three-species ecological network dynamics with two control inputs, we analyze the accessibility and controllability properties under the constraint that all species interaction coefficients a_{ij} lie in the open interval $(0, 1)$, representing non-negligible ecological interactions. The system also possesses invariant manifolds at coordinate planes where any species is absent, which restricts global controllability. A comprehensive mathematical analysis, including detailed computation of Lie brackets, rank verification, and examination of invariant sets, is provided in the Appendix in the Supplementary Information (SI).

The biological interpretation of these mathematical properties and their implications for control design in ecological management are discussed in the Results section.

3.1 Entropy as sustainability criteria

We identify sustainability with the idea that any action we take must not cause further depletion of resources. For example, if an action plan is designed such that one type of action is taken over a long period, it results in a cost-effective but not sustainable policy. Thus, we used Shannon entropy as an indicator of the degree of uniformity in the actions taken, i.e., we need to classify actions that use the maximum possible combinations in the control input space.

We have applied Shannon entropy to evaluate the degree of the sustainability of the control policies of the model. Shannon entropy is a fundamental information measure used to quantify the diversity in a distribution (Lin, 1991). It is mathematically defined as: $H = -\sum_{i=1}^{n} p_i \log\ (p_i)$, where p_i represents the proportion of data in the i-th bin, and n is the total number of bins (Shannon, 1948). The entropy reaches its minimum value, $H = 0$, when all the data is concentrated in a single bin ($p_i = 1$ for one bin and $p_j = 0$ for all $j \neq i$), indicating no diversity or uncertainty. Conversely, the entropy is maximized when the data is evenly distributed across all bins ($p_i = \frac{1}{n}$ for all i), in which case $H = \log(n)$, representing maximum diversity. For example, with six bins, the maximum entropy is $H = \log(6) \approx 1.7918$. Policies with higher entropy indicate a more robust application of interventions, such as non-monotone cutting of Lantana (u_1) and planting of control plant species (u_2), which ensures ecological stability and sustainability.

3.2 Numerical methods

We employ a model predictive control (MPC) framework to address the optimal control problem for managing the Lantana population dynamics defined by our three-species ecological system. The control objective is formulated as a variational problem that seeks to minimize a cost function while satisfying system dynamics and constraints. The cost function, given by

$$J = C_{\text{control}} \cdot \left(u_1^2 + u_2^2\right) + C_{\text{biodiversity}} \cdot \left((x_1 - x_{1d})^2 + (x_2 - x_{2d})^2 + (x_3 - x_{3d})^2\right), \tag{7}$$

balances two competing objectives: $C_{\text{control}} = 1000$ penalizes aggressive control efforts (harvesting u_1 and augmentation u_2), while $C_{\text{biodiversity}} = 10$ penalizes deviations from the desired ecological equilibrium state $(x_{1d}, x_{2d}, x_{3d}) = (0.1, 0.75, 0.15)$. The optimal control problem was implemented in Python using the CasADi optimization framework (Andersson et al., 2019). The nonlinear dynamics were discretized over a time horizon of $T = 25$ units using the fourth-order Runge-Kutta (RK4) method with step size $\Delta t = 0.03125$, divided into $N = 40$ control intervals. The discretized problem was formulated as a nonlinear programming (NLP) problem with state constraints maintaining populations within $[0.0, 1.0]$ and terminal constraints ensuring convergence to the target equilibrium. The NLP was solved using IPOPT (Wächter and Biegler, 2006). Control bounds were imposed as $u_1 \in [-5, 0]$ for harvesting the invasive species and $u_2 \in [0, 5]$ for augmenting native species, with two scenarios investigated: simultaneous application of both controls and selective harvesting only ($u_2 = 0$). The optimization was performed for various initial conditions, confirming successful convergence in all cases as shown in the results section.

We would like to point out that the form of the cost function chosen here is not exhaustive and one of the choice can be incorporating the entropy term which would require further detailed investigation.

4 Results

We provide solutions to the optimal control problem of the management of abundances of the invasive plant *Lantana camara*. We also analyzed the qualitative and quantitative behavior of these calculated optimal control policies, which can drive the system from any initial condition to a given desired state. We first present the results of analyzing the GLV network-based dynamical control systems model using Lie algebraic approaches. Secondly, by incorporating the principles of information theory into the model, we present Shannon entropy as a measure of the degree of sustainability of the control policies aligned with the Sustainable Development Goals (SDGs) paradigms.

4.1 Accessibility and controllability conditions

The controllability analysis of the three-species system (2) establishes the theoretical foundation for our optimal control framework and clarifies the extent to which desired system states can be reached. Under the ecological constraint that all interaction coefficients satisfy $a_{ij} \in (0, 1)$ for all $i, j \in \{1, 2, 3\}$ (Appendix, Assumption 1), representing non-negligible but bounded species interactions, the system is locally accessible at every point in the positive orthant $\mathbb{R}^3_{++}$ where all three species coexist (Appendix, Theorem 1). Importantly, this accessibility property holds uniformly across the entire parameter regime $a_{ij} \in (0, 1)$—it is not confined to specific parameter configurations but applies universally to all biologically realistic interaction strengths.

The proof involves computing Lie brackets $[f_0, f_1]$ and $[f_0, f_2]$ between the drift vector field (natural ecosystem dynamics) and control vector fields (management interventions), which reveal additional directions in state space accessible by combining natural dynamics with control actions. The accessibility distribution matrix $\mathcal{C}(x)$ (Appendix, Equation 12) has full rank equal to three throughout $\mathbb{R}^3_{++}$, satisfying the LARC. This accessibility depends critically on non-zero interaction coefficients a_{31} and a_{32}, representing how *Lantana camara* and native plants influence soil microbes (Appendix, Remark 1). These strictly positive coefficients provide the indirect control pathway: the control parameters u_1 and u_2 acting directly on plant populations can indirectly influence microbes through ecological coupling. If either of the coefficients a_{31} or a_{32} were zero, the accessibility matrix rank would reduce to 2, destroying controllability. Local accessibility guarantees that from any initial state $x_0 \in \mathbb{R}^3_{++}$, the reachable set has non-empty interior (Appendix, Theorem 2), meaning we can reach a neighborhood of any target state through appropriate control trajectories.

However, the system is not globally controllable. Forward-invariant manifolds exist at coordinate planes where a species has zero population (Appendix, Lemma 1, Theorem 4), and this limitation persists uniformly across all parameter values $a_{ij} \in (0,1)$. This reveals fundamentally different dependencies on parameters versus initial conditions.

Control efficacy is universal across the parameter regime, and accessibility holds for all values in $a_{ij} \in (0,1)$ without exception. However, control efficacy exhibits a sharp dichotomy with respect to initial conditions: if all species are initially present ($x_0 \in \mathbb{R}^3_{++}$), any target state can be approached.

We also investigated small-time local controllability (STLC) using Sussmann's sufficient condition. The system fails this condition uniformly across $a_{ij} \in (0,1)$ (Appendix, Theorem 3): the Lie brackets $[f_0,f_1]$ and $[f_0,f_2]$, essential for controlling microbes, cannot be expressed as combinations of control vector fields f_1, f_2, which span only the plant population plane. This failure is a structural feature persisting across all parameter values. While the condition is sufficient but not necessary, the failure suggests potential time-scale limitations on restoration speed (Appendix, Remark 3). Target states are eventually reachable from $\mathbb{R}^3_{++}$, but arbitrarily rapid transitions may not be achievable, aligning with ecological reality where restoration unfolds over extended periods. The optimal trajectories (Figures 2-4) reflect these natural time-scale constraints.

Hence, control efficacy is universal across the biologically realistic parameter regime $a_{ij} \in (0,1)$ and accessibility holds for all parameter values within this range, and controllability is not confined to specific parameter configurations. However, control efficacy is fundamentally constrained by initial conditions, i.e., desired states are reachable if and only if the initial state contains all three species at positive densities ($x_0 \in \mathbb{R}^3_{++}$). These results provide rigorous justification that computed optimal trajectories are theoretically achievable across diverse ecosystems with varying ecological characteristics, provided management begins early to maintain all species at positive densities throughout restoration. Given this analysis, we proceed to solve numerically the control problem and present the results in subsequent sections.

4.2 Comparative analysis of control strategies

Figure 5 presents a comprehensive comparison of system dynamics under different control scenarios. Figure 5a illustrates the uncontrolled state-space trajectories in the (x_1, x_2, x_3) for multiple initial conditions, showing the dismal state of Lantana invasion if no control is applied. Figure 5b depicts the controlled trajectories under two distinct control strategies: single control with $u_1 \neq 0, u_2 = 0$ (red curves) and dual control with $u_1 \neq 0, u_2 \neq 0$ (blue curves). Our simulations reveal that the case where only u_2 is active, while $u_1 = 0$, fails to drive the system to the desired state within the specified time horizon, indicating that the control input u_2 alone is insufficient for effective system stabilization. Hence, we have omitted this scenario. We observe that both the single-control strategy ($u_1 \neq 0, u_2 = 0$) and the dual-control strategy ($u_1 \neq 0, u_2 \neq 0$) successfully guide the system trajectories to the target state x_d for all initial conditions examined, but they demonstrate different control input signal structures. Figure 5c provides a quantitative comparison of the total costs associated with these two successful strategies, where the cost function balances state deviation from the target and control effort. The box plots reveal a striking difference: the single-control strategy incurs approximately double the cost (median $\approx$ 1300) compared to the dual-control strategy (median $\approx$ 700), demonstrating a substantial cost reduction of nearly 50%. The

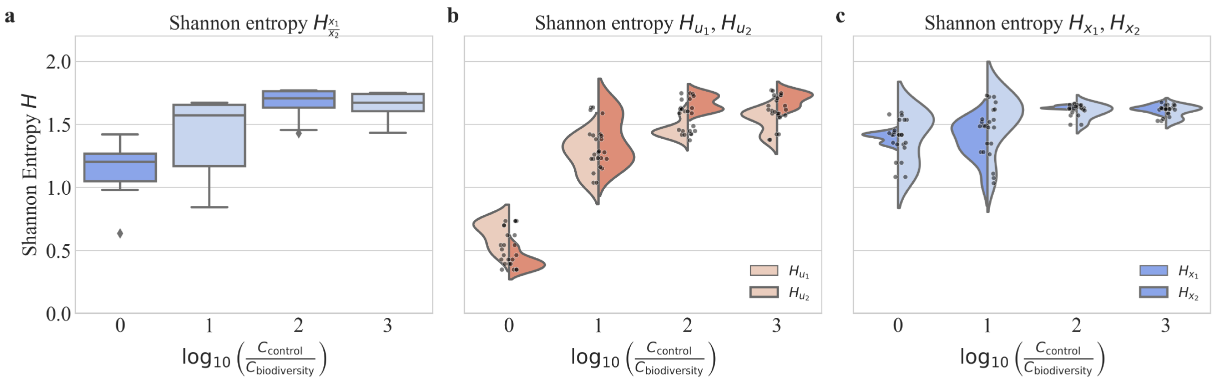

FIGURE 2
Shannon entropy H for evaluating the sustainability of control policies. **(a)** Box plot of H against $\log(c_{\text{control}}/c_{\text{biodiversity}})$ for the ratio of species abundances $\frac{x_1}{x_2}$, this gives the information about the distribution, in time, of the values of $\frac{x_1}{x_2}$ This indicates how this ratio behaves in time when initially it is $\frac{x_1}{x_2} > 1$ and crosses over to $\frac{x_1}{x_2} < 1$, across a set of varied initial conditions. **(b)** H_{u1} and H_{u2} from the distribution of the control signals in time for the same set of initial conditions as in **(a)**. **(c)** H_{x1} and H_{x2} for species abundance distribution in time for the same set of initial conditions as in **(a)**, reflecting control effectiveness and biodiversity outcomes.

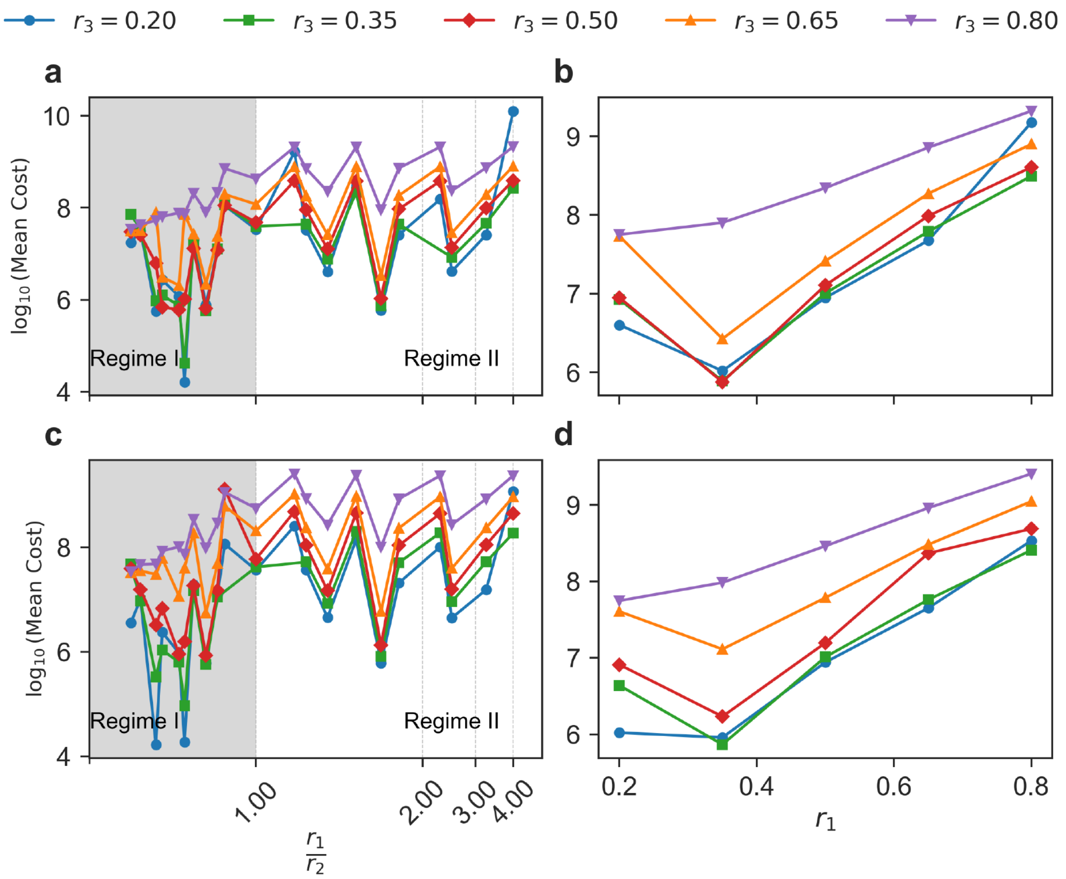

FIGURE 3

Sensitivity analysis: Mean control costs show an increasing trend with an increase in the growth rate of *Lantana*. **(a, b)** correspond to the initial condition population condition as $(x_1(0), x_2(0), x_3(0)) = (0.85, 0.1, 0.05)$ where we have a high abundance of Lantana. Figures **(c, d)** show results for a moderate abundance of Lantana with the initial condition $(x_1(0), x_2(0), x_3(0)) = (0.6, 0.3, 0.1)$. The interaction matrix is fixed as: $a_{11} = 0.3, a_{12} = 0.1, a_{13} = 0.4, a_{21} = 0.8, a_{22} = 0.2, a_{23} = 0.5, a_{31} = 0.6, a_{32} = 0.7, a_{33} = 0.7$. The mean is taken over for different values of the parameter r_2.

insets display representative time profiles of the control inputs, where inset (i) shows that in the single-control case, $u_1(t)$ must exhibit large variations to compensate for the absence of u_2, while inset (ii) illustrates that in the dual-control case, the control effort is distributed between $u_1(t)$ and $u_2(t)$. Collectively, these results establish that while the single-control strategy can stabilize the system, the dual-control policy provides superior performance by reducing intervention costs by approximately 50% and also producing smoother control actions that are more practical to implement in ecological management contexts, where resource constraints and minimized ecosystem disruption are critical considerations.

Since we find that the case of double controls incurs significantly lower cost than the single control one, all subsequent analyses are performed for the case of double control only.

4.3 Sustainability analysis using Shannon entropy

Selection of a sustainable policy requires a sieve, which we now elucidate. When designing control strategies for managing the invasive species *Lantana camara*, optimizing policies solely based on a fixed cost function is insufficient to ensure ecological sustainability. To address this limitation, we introduced Shannon entropy as an additional metric to evaluate and select control policies. Shannon entropy quantifies the diversity and balance of control efforts, providing insights into how evenly the management inputs are distributed over time. We show in Figure 4 how on changing the two parameters of the cost function, i.e., the control cost penalty parameter ($C_{Control}$) and the Bio-Diversity Penalty parameter ($C_{biodiversity}$) used in the optimization technique, we observed the different trajectories available in Figure 4. This also illustrates various optimal control policies, each with distinct ecological implications. For example, the policy in Figure 4 employs a non-monotone application of control inputs—cutting *Lantana camara* (u_1) and planting control plant species (u_2)—throughout the management period. This approach promotes a gradual replacement of *Lantana camara* with control plants, fostering ecological balance while minimizing disruptions to soil health, microbial communities, and the broader environment. In contrast, the policy in Figure 4 adopts a more aggressive strategy, concentrating intensive cutting and planting efforts at the beginning and end of the management period, with prolonged intervals of

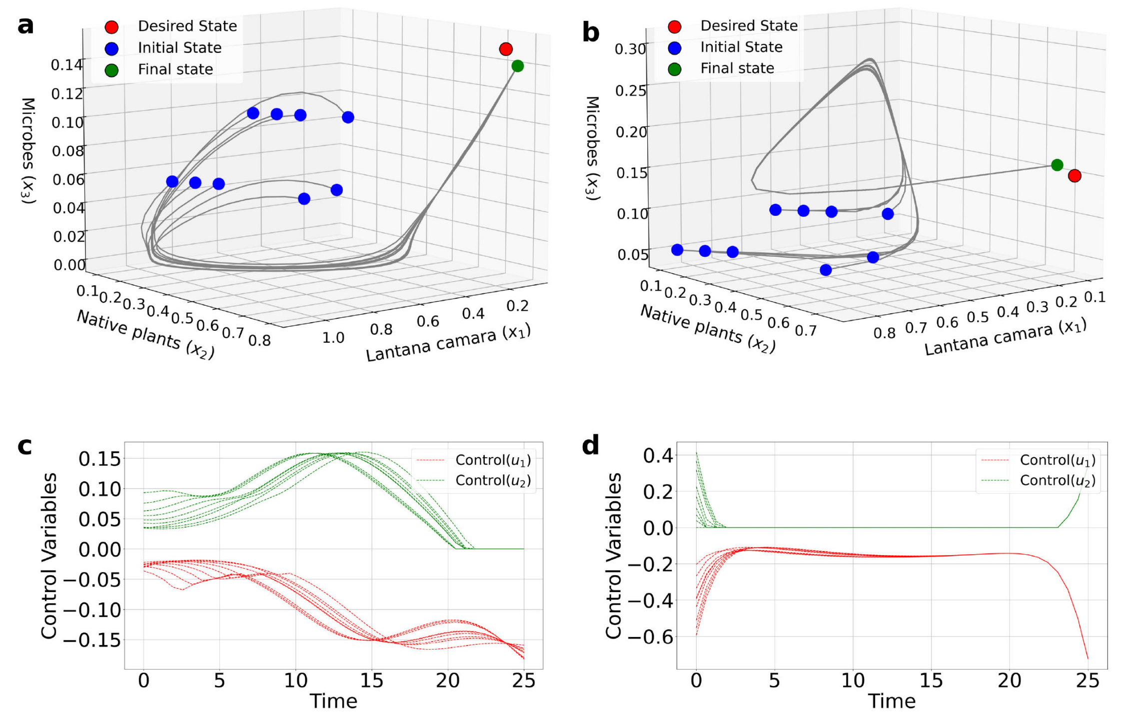

FIGURE 4
Control dynamics of the system across various initial species abundances. Growth rates: $r_1 = 0.85$, $r_2 = 0.4$, $r_3 = 0.3$. Desired state: $(x_1^d, x_2^d, x_3^d) = (0.1, 0.75, 0.15)$. Interaction matrix: $a_{11} = 0.6$, $a_{12} = 0.6$, $a_{13} = 0.5$, $a_{21} = 0.6$, $a_{22} = 0.3$, $a_{23} = 0.5$, $a_{31} = 0.7$, $a_{32} = 0.6$, $a_{33} = 0.7$. Cost parameters: **(a, c)**: Control cost $C_{control} = 1000.0$, biodiversity penalty $C_{biodiversity} = 1.0$. **(b, d)**: Control cost $C_{control} = 1000.0$, biodiversity penalty $C_{biodiversity} = 1000.0$.

inaction in between. The abrupt removal of *Lantana* without consistent planting risks leaving the land vulnerable to degradation, which can lead to soil erosion, nutrient depletion, microbial imbalance, and other cascading ecological issues. While this strategy may seem cost-effective, it compromises long-term stability and sustainability.

We report the impact of changes in the parameters of the cost function on the sustainability characterization of the feasible optimal control policies. Fixing the initial conditions as shown in Figures 5a, b and then using the defined cost function J in (Equation 7), and keeping the interaction matrix fixed(as given in the caption of Figure 4), we computed the optimal control trajectories

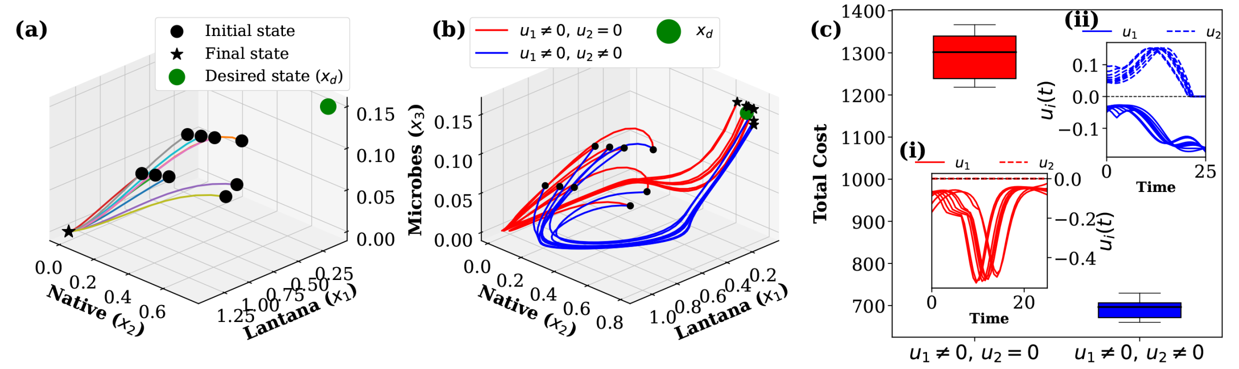

FIGURE 5
(a) Uncontrolled state-space trajectories in the (x_1, x_2, x_3) phase space converge away from the desired state (red cross), illustrating the prevalence of invasion dynamics in the absence of interventions. **(b)** Controlled trajectories under two strategies: red curves denote the case $u_1 \neq 0$, $u_2 = 0$, while blue curves denote $u_1 \neq 0$, $u_2 \neq 0$. Both control strategies steer the system closer to the desired state x_d (red cross), though with different transient dynamics. **(c)** Comparison of cumulative control costs between the two strategies (box plots), showing that simultaneous application of u_1 and u_2 yields substantially lower costs than the single-control case. Insets (i) and (ii) display the control input signals $u_1(t)$ (solid) and $u_2(t)$ (dashed) for two scenarios $u_1 \neq 0$, $u_2 = 0$ and $u_1 \neq 0$, $u_2 \neq 0$, respectively, highlighting distinct control effort distributions over time.

for various parameter pairs: $(C_{\text{control}}, C_{\text{biodiversity}}) = (10^3, 10^3)$, $(10^3, 10^2), (10^3, 10), (10^3, 1)$, across multiple initial conditions. For each trajectory, we have used six bins for discretization (We also found that on increasing the number of bins, the distribution did not change in shape). The Shannon entropy (H) was calculated for the time series of x_1, x_2, u_1, u_2, and the ratio of the Lantana and control plant abundances $\frac{x_1}{x_2}$. As can be seen in the distribution of the control inputs shown in Figure 2b, in the case of the example trajectory in Figure 4 corresponds to the ratio of $log_{10}\left(\frac{C_{control}}{C_{biodiversity}}\right) = 0$ having a lower value of Shannon entropy as compared to the trajectory in Figure 4 corresponding to the ratio of $log_{10}\left(\frac{C_{control}}{C_{biodiversity}}\right) = 3$ which has a higher entropy value. Based on these results and analysis, we report that there is a meaningful way to design a sustainable policy for a desirable ecological state by maximizing the Shannon entropy as the basis for selecting the action plan to control abundances. This trend promotes a more uniform distribution of control efforts over time, as indicated by higher entropy values. Policies with higher entropy condition align with ecological sustainability principles by ensuring continuous and evenly distributed management efforts, ultimately promoting a balanced and desirable resilient ecosystem.

4.4 Sensitivity analysis

We study sequentially how sensitive the proposed control policies are to the changes in model parameter values. There are two types of parameters regulating our proposed model: firstly, we have the dynamical systems parameters, and secondly, we have the parameters in the optimization of the control cost function. We checked and systematically analyzed the sensitivity of the control cost to both types of parameters, as shown in the previous sections. Now for the dynamical systems parameters, since the precise values of model parameters are unknown, we analyzed the trends of control costs by uniformly varying intrinsic growth rates (r_1, r_2, r_3) across the range $[0.2, 0.8]$, generating 125 tuples while keeping the values of the elements of the interaction matrix fixed. Simulations were conducted for two initial conditions: *Case I* as shown in Figures 3a, b, where, the abundance of the Lantana camara is high over the landscape $(x_1(0), x_2(0), x_3(0) = 0.85, 0.1, 0.05)$, and *Case II*, where its abundance is moderate $(x_1(0), x_2(0), x_3(0) = 0.6, 0.3, 0.1)$ as shown in the Figures 3c, d. The analysis revealed two distinct regimes based on the ratio of Lantana camara's growth rate (r_1) to that of the control plants (r_2), which matches the intuitive idea that we can use only certain plants to outgrow Lantana camara. In *Regime I*$(\frac{r_1}{r_2} < 1)$, where the growth rate of Lantana camara is lower than that of control plants, control costs are relatively low. In contrast, *Regime II* $((\frac{r_1}{r_2} > 1))$, where Lantana camara outcompetes control plants with higher growth rates, incurs significantly higher management costs, as evident in Figures 3a, c. These trends highlight the crucial role of competition dynamics, as higher costs emerge when Lantana camara dominates resource consumption and growth. Figures 3b, d further illustrate that the behavior of the mean (averaged over various values of r_2) control costs for scenarios of high and moderate Lantana camara abundances. The results clearly show that as r_1 increases, the management costs consistently rise. While the system shows limited sensitivity to the small variations within each regime, the absolute cost strongly depends on the relative growth rates and the initial abundance of the Lantana camara. These findings significantly highlight that the effective management of invasive species, such as Lantana camara, requires addressing growth dynamics and competition, as higher growth rates and competitive dominance significantly escalate management efforts to restore ecological balance.

5 Conclusion and discussion

Our study on optimal control strategies for managing the invasive plant *Lantana camara* highlights the crucial role of *Lantana camara*'s growth rate and competition with control plants in determining control costs, with two regimes identified: low costs when *Lantana camara*'s growth rate is lower than control plants, and high costs when *Lantana* outcompetes them. By integrating Shannon entropy as a metric for sustainability, we ensure that control policies are evenly distributed over time, promoting long-term ecological stability and minimizing risks like soil erosion and microbial imbalance. Sensitivity analysis reveals that control costs are influenced by both intrinsic growth rates and initial conditions, underscoring the necessity for adaptive, balanced management strategies.

The controllability analysis establishes that while the system is accessible at every point in $\mathbb{R}^3_{++}$ where all species coexist (uniformly across the parameter regime $a_{ij} \in (0, 1)$), it lacks global controllability due to forward-invariant manifolds at coordinate planes. The key finding is that control efficacy depends fundamentally on initial conditions rather than parameter regimes: our optimal control strategies are effective across diverse ecosystems with varying ecological characteristics, but only when applied before any species reaches local extinction. This emphasizes the critical importance of early intervention in invasive species management and maintaining minimal viable populations throughout restoration efforts.

The failure of Sussmann's sufficient condition for small-time local controllability suggests inherent time-scale limitations on restoration speed. While target states are eventually reachable from initial conditions in $\mathbb{R}^3_{++}$, arbitrarily rapid transitions may not be achievable, aligning with ecological reality where restoration unfolds over extended periods. The gradual optimal control trajectories (Figures 2-4) reflect these fundamental dynamical constraints rather than numerical artifacts. Additionally, the dramatic cost increase when *Lantana* growth substantially exceeds control plant growth (Figure 5) indicates the system approaching the controllability boundary where control plants become critically low, underscoring the practical significance of our entropy-based sustainability index in distributing control efforts temporally to prevent unintentional population collapse.

These findings suggest important extensions for future work. Integrating entropy directly into the objective functional represents a profound shift in perspective, moving the optimization framework from pure cost minimization (analogous to potential energy). This 'entropic type' functional, allows the formal control policy to balance the drive for efficiency and low cost against the necessity for action diversity and resource sustainability. The rigorous analysis of optimality conditions within this new formulation remains a significant and promising avenue for future theoretical research. Hybrid control strategies combining optimization with discrete species reintroduction events would address severely degraded sites where species are already absent. Incorporating stochasticity would account for parameter uncertainty and environmental noise, with chance constraints maintaining populations safely above zero while respecting invariant manifold limitations. Multi-timescale analysis could reveal whether exploiting fast microbial responses to steer slower plant dynamics improves controllability or reduces restoration time. Our controllability methodology also provides systematic tools for network expansion: Lie bracket calculations can predict whether adding species requires additional control variables, guiding experimental design for more complex multi-species management.

In conclusion, our integrated approach combining optimal control theory, entropy-based sustainability metrics, and rigorous controllability analysis ensures that control policies minimize costs while respecting fundamental dynamical constraints. The accessibility guarantees provide confidence that computed trajectories are achievable across diverse ecosystems provided management maintains all species at positive densities, while the identified limitations establish realistic boundaries for applicability and highlight where complementary approaches are necessary. This framework opens avenues for managing invasive species through plant-based interactions and provides tools for assessing control feasibility in expanded ecological networks.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material. Further inquiries can be directed to the corresponding author.

Author contributions

SK: Data curation, Visualization, Methodology, Conceptualization, Validation, Writing – original draft, Software, Formal analysis, Investigation, Writing – review & editing, Resources. PM: Resources, Methodology, Conceptualization, Validation, Visualization, Writing – original draft, Writing – review & editing, Formal analysis, Data curation, Investigation, Software. RS: Project administration, Validation, Methodology, Writing – review & editing, Investigation, Supervision, Writing – original draft, Conceptualization, Funding acquisition.

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Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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