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Asymptotic properties for self-weighted M-estimation of MEM

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Introduction: In the context of non-negative high-frequency financial time series, the multiplicative error model (MEM) is a generalized model. Maximum Likelihood Estimation (MLE) serves as the standard approach for parameter estimation when applying MEM to real-world modeling. This method relies on the assumption that the error term follows a specific known distribution with finite variance. However, directly imposing the assumption of a known distribution with bounded variance entails notable limitations.

Methods: In this model, a self-weighted M-estimation approach is used to estimate the model parameters. This estimation is performed considering the infinite variance of the model errors.

Results: On a theoretical level, this estimation proved to have strong consistency and asymptotic normality. The results of the numerical simulations show that self-weighted M-estimation is more robust than other estimation methods. Finally, the self-weighted M-estimation method is applied to the price range of polyethylene and polypropylene futures on the Dalian Commodity Exchange.

Discussion: The results demonstrate that the self-weighted M-estimation outperforms both maximum likelihood estimation and least absolute deviation estimation. This finding is particularly significant for financial applications, where extreme outliers and infinite-variance events are frequently observed.

KEYWORDS

asymptotic normality, consistency, multiplicative error model, price range, self-weighted M-estimation

1 Introduction

With the rapid advancement of computer intelligence technology, the use of digital models in financial markets has increased significantly in the 21st century. This convergence has led to remarkable advancements in the methods used to capture, process, and analyze financial data. The cost of recording and storing data has decreased significantly, resulting in favorable conditions for the collection, review, and examination of high-frequency financial data at intervals of only hours, minutes, or seconds. In general, increasing the frequency of data collection leads to a greater amount of market information being collected and less information being lost. This is of particular importance for the systematic investigation of the market and market conditions. Building on the ARCH class models, Engle proposed a new model called the multiplicative error model (MEM) for non-negative financial time series. MEM serves the purpose of characterizing the fluctuation features of sequences with non-negative values. In addition, MEM addresses the limitations of existing models that do not account for non-negativity throughout the modeling process.

A non-negative process can be represented as the combination of a positive random variable and a time-varying scaling factor. Let $\{x_t, t \in N\}$ be a non-negative financial time series, and let $\mathcal{F}_{t-1} = \sigma\left(\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\right)$ represent the information set available up to the time period $t-1$. Then, the standard MEM$\left(p, q\right)$ is a model of a conditional expectation equation with a specific order, which can be represented as follows:

$$x_t = \mu_t \varepsilon_t, \tag{1}$$

$$\mu_t = \omega + \sum_{i=1}^{p} \alpha_i x_{t-i} + \sum_{j=1}^{q} \beta_j \mu_{t-j}, \tag{2}$$

Among them, $\mu_{\mathbf{t}}$ is the conditional expectation of x_t based on $\mathcal{F}_{\mathbf{t-1}}$, $\varepsilon_{\mathbf{t}}$ is a positive random error term with a mean of 1, and the coefficients in the model satisfy $\omega > 0$, $\alpha_i \geq 0 \left(i = 1, 2, \ldots, p\right)$, and $\beta_j \geq 0 \left(j = 1, 2, \ldots, q\right)$, where p and q are the lag order of the model.

Engle [1] proposed MEM under the condition that the error term follows an exponential distribution and briefly introduced the properties and estimation methods of this model. Subsequently, Lanne [2] proposed a mixed MEM, and in the empirical analysis, the error term was in the form of a mixed gamma distribution, referred to as GMEM in this case. The results of the analysis revealed that the fitting and prediction results under the mixed gamma distribution were superior to GARCH-type models [2]. Afterward, many scholars extended the MEM in various aspects, some proposing methods, such as EMEM and WMEM, have focused on the Weibull distribution of error terms (see Hong [3] for details), while others have extended the model form. For example, Lam and Ng [4] proposed the MEM-GARCH model, while Zhou and Zhang [5] proposed a relative error estimation method for MEM based on the least squares criterion. The simulation results show that this method has certain advantages compared to other similar methods. Li [6] proposed the threshold vector multiplicative error model (TVMEM), and also the smooth transformation vector multiplicative error model (STVMEM) with Li and Lu [7]. Lu et al. [8] introduced two robust estimating methods, M-estimates and BM-estimates, for the multiplicative error model, and they analyzed the impact of different loss functions on the estimation results of these methods. Lu and Shi [9] introduced a model-averaging estimation approach for the multiplicative error model. The existing parameter estimation methods of the multiplicative error model can theoretically achieve second-order unbiased, but the precision of uncertainty can only achieve first-order unbiased. Wang and Zou [10] used a new method to improve the accuracy of uncertainty.

2 Materials and methods

Maximum likelihood estimation (MLE) is the standard method for estimating the parameters when using MEM for real-world modeling. It is based on the idea that the error term follows a known distribution with limited variance. However, the direct assumption that the error term follows a known distribution with bounded variance has obvious limitations. The high-frequency data observed in real financial markets often exhibit heavy-tailed characteristics and tend to contain multiple outliers. Moreover, the variance of such data can potentially be infinite. Therefore, it is unreasonable for MLE to explicitly assume that the error variance during parameter estimation is bounded. When the previously expected error distribution does not match the actual scenario, the results obtained will be incorrect. Given the challenges posed by maximum likelihood estimation (MLE), it is imperative to improve the optimization of the technique for estimating the model parameters. Both theoretical and empirical studies demonstrate that MLE lacks robustness when sample data follow heavy-tailed distributions or contain outliers, resulting in significant deviations from the true values. To address this issue, Huber [11] proposed M-estimation as a robust estimation method. Assume a sample $\{X_1, X_2, \cdots, X_N\}$, the least squares estimation method seeks the parameter μ that minimizes $\sum_{t=1}^{N} \left(X_t - \mu\right)^2$. However, since sample data often contain outliers, the least squares estimator is not robust. Therefore, Huber proposed replacing the quadratic function in the least squares estimation with an alternative function of the sample errors, that is, finding the parameter μ that minimizes the following expression

$$\sum\nolimits_{t=1}^{N} \rho\left(X_t - \mu\right), \tag{3}$$

where $\rho\left(\cdot\right)$ denotes the loss function, and the resulting statistic is generally referred to as an M-estimation. Consequently, the primary objective of this study is to select an appropriate loss function $\rho\left(\cdot\right)$ that enables the MEM parameters to achieve robust estimation, making it suitable for heavy-tailed data characterized by outliers. However, financial markets are susceptible to shocks arising from new information, which often generate outliers or leverage points. It is therefore unreasonable for M-estimation to assign equal weight to these outliers as it does to normal observations. Therefore, this study considers SM-estimation—which assigns distinct weights to outliers based on their magnitude—to further mitigate their influence on estimation results; that is, we seek to find μ the that minimizes the expression

$$\sum\nolimits_{t=1}^{N} w_t \rho\left(X_t - \mu\right), \tag{4}$$

where $w_t = w\left(x_{t-1}, x_{t-2}, \cdots\right)$ is the weight function.

The theoretical features and applications of this estimator are also examined. The structure of this article is as follows: After introducing some key concepts, the primary results of this study (asymptotic normality and strong consistency) are discussed in Section 3. The results of the numerical simulation and the empirical analysis of the SM-estimation are presented accordingly in Section 4. The conclusion is presented in Section 5. Finally, the method used to verify the results is described in the Appendix.

3 Results

Let $\theta = \left(\omega, \alpha_1, \cdot\cdot\cdot, \alpha_p, \beta_1, \cdot\cdot\cdot, \beta_q\right)^T$ be the model parameter, and from Equation 2, μ_t is related to θ, so it can be recorded as $\mu_t = \mu_t\left(\theta\right)$. Taking the logarithm at both ends of Equation 1 yields

$$ln x_t = ln \mu_t\left(\theta\right) + ln \varepsilon_t, \tag{5}$$

Let $y_t = \ln x_t - c_0$, $\eta_t = \ln \varepsilon_t - c_0$, and $c_o = median\left(\ln \varepsilon_t\right)$, then

$$y_t = ln\mu_t\left(\theta\right) + \eta_t, \tag{6}$$

Therefore, the SM-estimation of parameters in MEM(p,q) is

$$\tilde{\theta}_{SM} = \arg\min_{\theta\in\Theta} \sum_{t=v+1}^{n} w_t \rho\left(y_t - \ln \mu_t\left(\theta\right)\right), \tag{7}$$

where $v = \max\left(p,q\right)$, $\Theta \subset R^{p+q+1}$ represents the parameter space, $w_t = w_t\left(x_{t-1}, x_{t-2}, \cdots\right) > 0$ is an optional weight function, and $\rho\left(\cdot\right)$ represents a non-negative loss function.

According to Engle and Russell [12], if sequence x_t satisfies the condition of stationarity, then $\mu_t\left(\theta\right)$ can be represented as

$$\begin{aligned}\mu_t\left(\theta\right) = {} & \frac{\omega}{1-\sum_{j=1}^{q}\beta_j} + \sum_{i=1}^{p}\alpha_i x_{t-i} \\ & + \sum_{i=1}^{p}\alpha_i \sum_{k=1}^{\infty}\sum_{j_1=1}^{q}\cdots\sum_{j_k=1}^{q}\beta_{j_1}\cdot\cdot\beta_{j_k}x_{t-i-j_1-\cdots-j_k},\end{aligned} \tag{8}$$

Let us denote $U_{t0}\left(\theta\right) := \frac{\partial \ln \mu_t(\theta)}{\partial \omega}$, $U_{ti}\left(\theta\right) := \frac{\partial \ln \mu_t(\theta)}{\partial \alpha_i}$, $U_{t(p+j)}\left(\theta\right) := \frac{\partial \ln \mu_t(\theta)}{\partial \beta_j}$, $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, q$, $U_t\left(\theta\right) := \frac{\partial \ln \mu_t(\theta)}{\partial \theta} = \left(U_{t0}\left(\theta\right), U_{t1}\left(\theta\right), \cdots, U_{t(p+j)}\left(\theta\right)\right)^T$, θ_0 is the true value for θ.

In order to obtain the asymptotic properties of the SM-estimation, we provide the following assumptions.

Assumption 3.1 Let θ_0 be the true parameter vector of the model, where θ_0 is both greater than 0 and lies in an interior point in Θ, and for each $\theta \in \Theta$, $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$.

Assumption 3.2 ε_t is independent and identically distributed, with $E|\varepsilon_t| < \infty$.

Assumption 3.3 $\rho(\cdot)$ is a non-negative convex function, $\rho(0) = 0$, and there is a derivative ψ .

Assumption 3.4 The distribution function of η_t is $F(x)$, the density function is $f(x)$ and differentiable everywhere, with $f(x) > 0$ and $\sup_{x\in R}\left|f'\left(x\right)\right| < \infty$.

Assumption 3.5 w_t is a positive value function and satisfies $E(w_t + w_t^2)\left\|U_t(\theta)\right\|^2 < \infty$, $\|\cdot\|$ denotes the Euclidean norm of a vector.

Assumption 3.6 Assuming $G(t) := E\psi(\eta_1 + t)$ exists, and its derivative when $t = 0$ is $\lambda > 0$, with $G(0) = 0$.

Remark 3.1 Assumptions 3.1 and 3.2 are commonly used in MEM research, which ensure the stationarity and ergodicity of the sequence $\{x_t\}$ and $Ex_t < \infty$. Assuming 3.3 restricts the function $\rho\left(\cdot\right)$, if the derivative of $\rho\left(\cdot\right)$ is bounded, then the estimation is robust to the heavy-tailed distribution; if $\rho\left(\cdot\right)$ is also bounded, then the estimation is robust to outliers. As for the weight function in assumption 3.5, $w_t = 1$ can be chosen when the data properties are good; when the data properties are poor (with infinite variance or outliers), $w_t = \left(1 + 5\left(x_{t-1} + \cdots + x_{t-p}\right)^3\right)^{-1}$ can be chosen to reduce the impact of outliers. For more information on the form of weight functions, we can see Davis and Dunsmuir [13] and Ling [14].

Next, we present two main results of this study.

Theorem 3.1 *Considering the standard MEM(p,q) defined by Equations 1, 2, under Assumptions 3.1–3.3 hold, as $n \to \infty$,*

$$\tilde{\theta}_{SM} \to \theta_0 \quad a.s., \tag{9}$$

where a.s. is short for "alomst surely" convergence.

Theorem 3.2 *If Assumptions 3.1–3.6 hold, then as $n \to \infty$,*

$$\sqrt{n}\left(\tilde{\theta}_{SM} - \theta_0\right) \xrightarrow{d} N\left(0, \frac{\Sigma^{-1}\Omega\Sigma^{-1}}{4f^2\left(0\right)}\right) \tag{10}$$

where $\Sigma = E(\mathbf{w_t}U_t(\theta_\mathbf{0})U_t(\theta_\mathbf{0})^\mathrm{T})$, $\Omega = E(\mathbf{w_t^2}U_t(\theta_\mathbf{0})U_t(\theta_\mathbf{0})^\mathrm{T})$, $\xrightarrow{d}$ means convergence in distribution.

Remark 3.2 Lu and Ke [15] have demonstrated the asymptotic normality of M-estimation, while Theorems 3.1 and 3.2 provide strong consistency and asymptotic normality of SM-estimation under the condition of infinite error variance, which has wider applications. This generalization extends previous conclusions in the literature and demonstrates that the proposed estimator has broader applicability, particularly in financial contexts characterized by extreme volatility.

4 Numerical simulation and empirical analysis

In this section, a finite sample simulation study is conducted for the SM-estimation of parameters. It primarily compares the performance of SM-estimation with MLE and M-estimation when the error contains certain outliers. In M-estimation, outliers and normal points in the data are weighted equally, which may be somewhat unreasonable. In SM-estimation, outliers can be weighted differently depending on their size, which further reduces the impact of outliers on estimation results. Consider the standard MEM$(1,1)$, where the true value of θ is set to $(0.2, 0.3, 0.5)$,and the error ε_t is assumed to follow four distributions with expected values of 1, Exp(1), Pareto(3, 2), Burr(3, 2.2, 0.6), and Fréchet(1.5, 0.37), respectively. We know that financial market transactions are frequent and that new information can emerge at any time that can affect market transactions. High-frequency data are more sensitive to the reflection of market information, and abnormal points often appear in high-frequency financial sector data. To obtain a better picture of how high-frequency financial data are generated, 10% of the data are randomly selected from the error sequence of random numbers and added with three times the standard deviation of the distribution as outliers. In SM-estimation, select

$$w_t = \begin{cases} 1, a_t = 0, \\ \frac{c^3}{a_t^3}, a_t \neq 0, \end{cases} \tag{11}$$

where $a_t = |x_{t-1}|\, I\,(|x_{t-1}| \geq c)$, c is the 90% quantile point of the sequence x_t.

In this study, we choose the Huber function as the loss function in the SM-estimation, which is

$$\rho_k(x) = \begin{cases} \frac{1}{2}x^2, |x| \le k, \\ k|x| - \frac{1}{2}k^2, |x| > k, \end{cases} \qquad (12)$$

where k is to be determined. Following Huber (11), we set $k = 1.345$. This value ensures that the SM-estimation achieves 95% efficiency relative to the MLE under normality, while providing robustness to outliers.

To ensure the full reproducibility of the simulation, generate observation data with a sample size of 500 and compare the average deviation (Bias) and mean square error (MSE) of the three estimation methods after 2,000 repetitions. All simulations are performed in R. The specific results are shown in Table 1.

From Table 1, it can be observed that the S-Huber estimation results are generally better than Least Absolute Deviation (LAD) and MLE in terms of both light-tailed and heavy tailed error distributions. This indicates that the results of SM-estimation are more robust and suitable for modeling financial data with heavy tails and outliers (similar simulation results can also be obtained for higher-order models such as MEM$(1,2)$, MEM$(2,1)$, and MEM$(2,2)$, indicating that the results of SM-estimation are more robust. This section is limited to space and does not include numerical simulation results for higher-order models.

To demonstrate the effectiveness of the SM-estimation method in practical applications, we selected two large, liquid, and industry-representative 5-min high-frequency futures trading datasets from the Dalian Commodity Exchange. Specifically, the samples for empirical analysis consist of price range data for polyethylene and polypropylene from 1 August 2017 to 30 April 2019. Before analyzing the data, it is necessary to preprocess the original data: the price range is the highest price minus the lowest price within each 5-min trading interval. Table 2 presents the descriptive statistical analysis results of the price range of two futures.

From Table 2, it can be observed that the average price range of polyethylene and polypropylene is 18.11 and 18.85, respectively, with standard deviations of 10.90 and 11.75. These two indicators indicate that the trading activity of polyethylene is greater than that of polypropylene. When looking at skewness, both polyethylene and polypropylene have skewness values greater than 0, which indicates that the price range distribution curves for both are positively skewed, making them good for modeling non-negative models. From the perspective of kurtosis, the kurtosis

TABLE 1 Simulation results under different distributions.

Distributions	Estimations	Bias (ω)	Bias (α)	Bias (β)	MSE (ω)	MSE (α)	MSE (β)
Exp(1)	MLE	0.0816	0.0861	−0.0101	0.0122	0.0109	0.0048
	LAD	0.0222	−0.0036	−0.0119	0.0086	0.0041	0.0112
	SLAD	0.0204	−0.0008	−0.0118	0.0086	0.0051	0.0117
	Huber	0.0096	−0.0174	−0.0100	0.0052	0.0031	0.0079
	S-Huber	0.0079	−0.0148	−0.0101	0.0052	0.0037	0.0083
Pareto$(3,2)$	MLE	0.1250	0.1459	−0.0127	0.0285	0.0319	0.0057
	LAD	0.0181	0.0006	−0.0114	0.0060	0.0044	0.0080
	SLAD	0.0166	0.0046	−0.0120	0.0062	0.0058	0.0089
	Huber	0.0150	−0.0028	−0.0089	0.0048	0.0035	0.0063
	S-Huber	0.0129	0.0009	−0.0087	0.0048	0.0048	0.0071
Burr $(3,2.2,0.6)$	MLE	−0.0754	−0.1659	−0.1732	0.0089	0.0297	0.0904
	LAD	0.0361	0.0064	−0.0693	0.0211	0.0161	0.0825
	SLAD	0.0366	0.0100	−0.0714	0.0218	0.0188	0.0855
	Huber	0.0363	−0.0054	−0.0751	0.0209	0.0126	0.0827
	S-Huber	0.0337	−0.0021	−0.0712	0.0206	0.0145	0.0829
Fréchet $(1.5,0.37)$	MLE	0.4383	0.2743	−0.0130	50.2703	0.4308	0.0090
	LAD	0.0188	−0.0010	−0.0028	0.1956	0.0019	0.0034
	SLAD	0.0175	0.0013	−0.0027	0.1950	0.0027	0.0039
	Huber	0.0348	0.0248	−0.0017	0.0591	0.0023	0.0025
	S-Huber	0.0340	0.0274	−0.0023	0.0591	0.0032	0.0030

TABLE 2 Descriptive statistical analysis of futures price range.

Futures name	Sample size	Mean	Std	Skewness	Kurtosis	JB stat	Ljung-Box Q(10)
Polyethylene	19,650	18.11	10.90	2.79	18.43	313,832 (0.0000)	10,216 (0.0000)
Polypropylene	19,650	18.85	11.75	2.93	17.58	281,324 (0.0000)	7,590.4 (0.0000)

TABLE 3 Comparisons of three estimations.

Futures name	Estimations	ω	α	β	*LogL*	*AIC*
Polyethylene	S-Huber	1.695	0.261	0.656	−1.775	3.551
	LAD	1.616	0.274	0.637	−1.794	3.589
	MLE	1.774	0.263	0.639	−1.794	3.588
Polypropylene	S-Huber	1.789	0.229	0.687	−1.862	3.725
	LAD	1.900	0.250	0.652	−1.878	3.756
	MLE	1.724	0.242	0.669	−1.878	3.756

of both polyethylene and polypropylene is also greater than 3, and the kurtosis of polyethylene is greater than that of polypropylene. This indicates that the distribution curves of both exhibit obvious peak-thick tail characteristics, and the peak-thick tail characteristics of polyethylene are more pronounced than those of polypropylene. In addition, from the results of the Jarque-Bera (JB) statistic in the table, it can be observed that the P-values of both JB tests are 0, rejecting the null hypothesis of normality and indicating that both are non-normal distributions. Using MLE to establish BMEM$(1, 1)$ for the price range of two futures (compared to models such as EMEM, BMEM estimation is relatively effective), and using M-estimation and SM-estimation to estimate MEM$(1, 1)$ parameters. The specific results are shown in Table 3.

From Table 3, it can be observed that $\alpha + \beta$ of both futures is less than 1, meeting the requirements for model stationarity, and the values are both greater than 0.85, indicating a strong clustering effect. From the results of LogL, the LAD estimation results are relatively close to those of MLE. From the results from MLE to S-Huber estimation of AIC, polyethylene decreases from 3.588 to 3.551, with a decrease of 1.04%; polypropylene decreases from 3.756 to 3.725, with a decrease of 0.83%. Overall, the S-Huber estimation results are better than MLE's. The autocorrelation test is conducted on the residual $(\frac{x_t}{\hat{\mu}_t})$ after applying S-Huber estimation. The Ljung Box Q statistics for two futures at lags of 10 and 20 orders, evaluated at a significance level of 5%, are 13.809 (0.055), 18.619 (0.351), 12.151 (0.096), and 17.065 (0.450), respectively. Both accepted the original hypothesis, indicating that the residuals meet the condition of being independent and identically distributed and that the model fitting is relatively sufficient.

The Hill estimation method introduced by Resnick [16] was used to estimate the tail index of the residual sequence. Figures 1, 2, respectively, show the tail index estimation of the residual sequence after modeling the price range of polyethylene and polypropylene using S-Huber estimation. We cannot conclude from the figure that the variance of the residual sequence is finite, which indicates that the results of SM-estimation are more reliable compared to others.

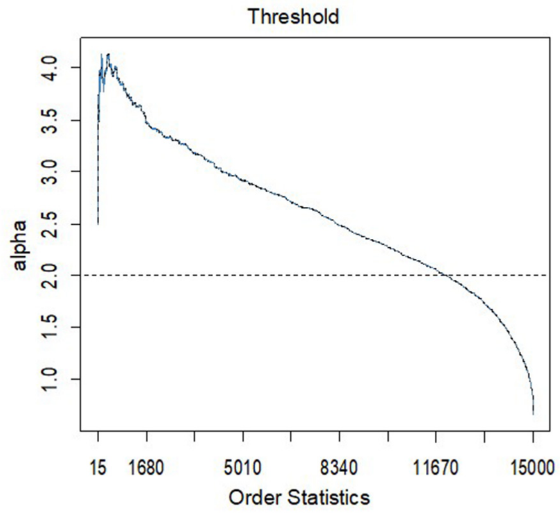

FIGURE 1
Hill estimation of polyethylene price range.

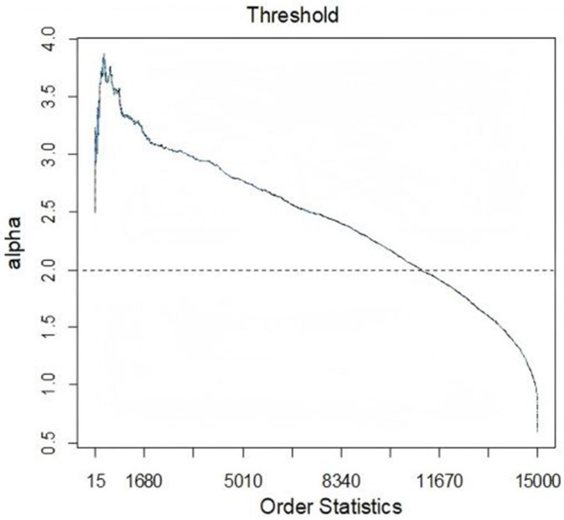

FIGURE 2
Hill estimation of polypropylene price range.

5 Conclusion

This study addresses the limitations of maximum likelihood estimation (MLE) in this context by introducing a self-weighted M-estimation approach. While MLE is conventionally used under the assumption that error terms follow a known distribution with bounded variance, this premise often fails in real-world scenarios. Real financial high-frequency data typically exhibit heavy-tailed characteristics and multiple outliers, implying potentially infinite variance. Consequently, relying on MLE under strict distributional assumptions can lead to model misspecification and inaccurate results when the actual error distribution deviates from expectations. In contrast, the self-weighted M-estimation proposed in the study explicitly accommodates the possibility of infinite variance in model errors.

On a theoretical level, we establish that the self-weighted M-estimator possesses strong consistency and asymptotic normality. Numerical simulations further demonstrate that this method exhibits greater robustness compared to other estimation methods, particularly in the presence of heavy tails. To validate practical applicability, we apply the proposed method to the price range of polyethylene and polypropylene futures on the Dalian Commodity Exchange. The empirical results reveal that the self-weighted M-estimation outperforms competing methods, achieving a superior fit.

In summary, the self-weighted M-estimation approach overcomes the fragility of MLE in the face of heavy-tailed distributions and infinite variance. These findings offer a more robust and reliable tool for modeling high-frequency financial processes where infinite variance is a concern.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

TL: Writing – original draft, Writing – review & editing. SH: Writing – review & editing, Supervision. K-AF: Writing – review & editing.

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The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fams.2026.1764133/full#supplementary-material

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