frontiers | Frontiers in Applied Mathematics and Statistics

TYPE Original Research
PUBLISHED 04 February 2026
DOI 10.3389/fams.2026.1744058

OPEN ACCESS

EDITED BY
Appanah Rao Appadu,
University of the Western Cape, South Africa

REVIEWED BY
Yannick Tangman,
University of Mauritius, Mauritius
Faiza Sami,
Govt Gordon Graduate College
Rawalpindi, Pakistan

*CORRESPONDENCE
Zakariya Yahya Algamal
✉ zakariya.algamal@uomosul.edu.iq

RECEIVED 11 November 2025
REVISED 09 January 2026
ACCEPTED 12 January 2026
PUBLISHED 04 February 2026

CITATION
Basheer GT, Waleed Mahmood S and Algamal ZY (2026) Improving parameters estimation of a truncated Poisson regression model based on meta-heuristic optimization algorithms.
Front. Appl. Math. Stat. 12:1744058.
doi: 10.3389/fams.2026.1744058

Improving parameters estimation of a truncated Poisson regression model based on meta-heuristic optimization algorithms

Ghalya Tawfeeq Basheer[1], Shaimaa Waleed Mahmood[2] and Zakariya Yahya Algamal [2]*

[1]Department of Operations Research and Intelligent Techniques, University of Mosul, Mosul, Iraq, [2]Department of Statistics and Informatics, University of Mosul, Mosul, Iraq

The paper discusses computational and numerical challenges that are associated with the truncation of the information and which change the usual Poisson likelihood by the introduction of black kite optimization algorithm. Real data is used to demonstrate a significant improvement in healthcare and medical research. Truncated Poisson regression models (TPRM) are essential for analyzing count data where zero counts are unobserved, a common scenario in many real-world applications, such as healthcare and medical research. However, parameter estimation in such models often suffers from bias and inefficiency due to the complexity induced by truncation. This study proposes an improved parameter estimation approach for TPRM by leveraging meta-heuristic optimization algorithms. Specifically, we integrate state-of-the-art meta-heuristics, black kite optimization algorithm (BKA), to optimize the likelihood function and overcome the limitations of traditional iterative methods such as Newton-Raphson and quasi-Newton algorithms. Using extensive Monte Carlo simulations and real data application, we evaluate the performance of the proposed method under varying sample sizes and covariate structures. The results demonstrate that our meta-heuristic-based estimator significantly reduces mean squared error compared to conventional estimators, enhancing model reliability and predictive accuracy. The proposed approach offers a robust and efficient alternative for parameter estimation in truncated Poisson regression, with potential applications in epidemiology, ecology, and other fields dealing with truncated count data.

KEYWORDS

BFGS, black kite optimization algorithm, count data, meta-heuristic optimization algorithms, truncated Poisson regression model

1 Introduction

Count data modeling is a statistical field of study directed at such variables as dependent variables that are identifications of a non-negative integer number of events or occurrences, i.e., the quantity of visits to a doctor, traffic incidents, or children within a family [1, 2]. Leading to the right skewed distribution, among other characteristics unlike continuous outcomes of count data are discrete that tend to have a distorted distribution centered on lower values with a high proportion of zeros [3].

Poisson regression, a classical modeling strategy of the count data, presupposes the idea of the counts being distributed according to Poisson distribution with its means (equidispersion). This model presents the expected count as an exponential of explanatory variables that makes its interpretation to be simple in the generalized linear model framework. Empirical data however, generally do not fit the equidispersion assumption, therefore assuming they are over dispersed, which is very common, or under dispersed, which is rare. Such cases are dealt with by substituting such models with newer models with more parameters such as the Negative Binomial regression that has an extra parameter to fit flexibility to variance independent of the mean. Other distributions intended to handle different dispersion are the Generalized Poisson Conway-Maxwell-Poisson (COM-Poisson) and similar distributions [4, 5].

The other difficulty which is commonly seen is the occurrence of excess zeros; that is more counts than are predicted by standard count models. At the cost of this, zero-inflated models and hurdle models have been designed. The two components have independent structure each component modeling the result on the binary zero-non zero and the count distribution of the positive values, and thus generates better fitting and inference in situations where zeros are produced by separate mechanisms [6, 7].

The truncated Poisson regression model is a statistical technique used to analyze a count-type of data that are truncated; such as dynamic observations are excluded in the sample when smaller or larger than some integer limits [8, 9]. Truncated Poisson regression contrasts with the usual Poisson regression model in that the counts take a truncated distribution due to the requirement that the counts exceed (left truncation) or are less than (right truncation) a given value [10]. This method would consider the fact that the realized counts do not fall into the grand scope of counts that might be observed hence avoid bias in parameter estimation and wrong inference that would have been made had truncation not been considered [11, 12].

Optimization means achieving the best result under certain conditions. Engineers must make many technical and managerial decisions throughout the design, construction, and upkeep of any engineering system. The main purpose of all these decisions is to either minimize the effort or to maximize the benefit. In any situation, the effort required or the amount of benefit can be defined as a function of certain decision variables. Optimization is then the process of choosing the conditions that result in the highest or lowest value of a function [13, 14].

There is no one way that can solve all optimization problems efficiently. Due to this, various optimization methods have been invented to handle many types of optimization problems. These methods are also called mathematical programming and are typically included in the field of operations research. Operations research uses scientific methods to solve problems related to decision making and to find the best solutions [13, 14]. In recent years, new optimization methods have gained popularity and are widely used to solve difficult engineering optimization problems. Such as genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, black-winged kite optimization algorithm [13].

The black kite optimization algorithm (BKA) is a meta-heuristic optimization algorithm based on the migration and hunting patterns of the black kite. The BKA brings together the Cauchy mutation technique and the Leader strategy to increase its ability to find global solutions and speed up convergence. This method manages achieves a balance both global and local knowledge to find good solutions [15]. It should be noted that not all problems can be solved by one algorithm. There is no meta-heuristic algorithm that is optimal in all optimization problems. A meta-heuristic algorithm can do very well with certain types of issues, but may not be as successful with others. The advancement of technology and the rise in problem complexity make it hard for some traditional algorithms to solve them [15].

The key shortcoming of parameter estimation in the truncated Poisson regression models can be viewed as the complex effect of the likelihood that is induced by truncation as it may necessitate the employment of the non-linear optimization tools and cause other problems with acquiring consistent and efficient estimators. Truncated Poisson models can be estimated using maximum likelihood in which the likelihood functions are complex with the normalizing constant being dependent upon the truncation limits. That increases the computational burden of parameter estimation and frequently requires non-linear optimization techniques iterative in nature, which have the potential to converge slowly or at local as opposed to global optima [16].

The weaknesses of quasi-Newton algorithms in parameter estimation of the truncated Poisson regression model mostly follow these issues because of the difficulty of the truncated likelihood function, which influence convergence and speed of computation. In the truncated Poisson measuring and computing log-likelihood, the log-likelihood contains a normalizing constant that relies on the truncation that results in the likelihood function to be non-linear and consequently more strenuous compared with that in the conventional Poisson. Quasi-Newton optimization methods do not compute the Hessian explicitly but can become sensitive to this complexity and thus slow to converge or to find global (as opposed to local) maxima. Quasi-Newton optimization can be sensitive to starting parameter values, and is iterative, whereas truncation or small sample size limits often render it insensitive to such start values. Such sensitivity may induce instability or convergence to irreliable parameter estimates [17, 18].

The paper therefore concerns the difficulty of causing the parameters of the truncated Poisson regressions to get the accurate estimation of parameters based on maximum likelihood approach algorithms. In particular, the paper discusses computational and numerical challenges that are associated with the truncation of the information and which change the usual Poisson likelihood by the introduction of black kite optimization algorithm. The complexity typically gets in the way of rapid convergence, arithmetical precariousness and can be biased when using traditional optimization schemes like Newton-Raphson, Gauss-Newton, or perhaps quasi-Newton procedures.

2 Poisson regression

The modeling of count data is based on Poisson regression. It was the earliest model to explicitly model counts and it remains at the foundation of the numerous kinds of count models that can be used by analysts, it is also called log-linear regression. The dependent variable in this model is Poisson distributed and the dependent variable is linked with the independent variables using a log link function which yields a linear equation [4, 6].

The Poisson regression model states that the y_i is a random variable distributed as Poisson with parameter λ_i, which depends on the regressors x_i. The main formula of the model is [19]:

$$P\left(Y_i = y_i \,|x_i\,\right) = \frac{e^{-\lambda}\lambda^{y_i}}{y_i!}, \;\; y_i = 0, 1, 2\ldots, \tag{1}$$

The most popular formulation for λ_i is the loglinear model

$$ln\lambda_i = x_i^T\beta \tag{2}$$

where $x_i^T\beta = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_n x_{in}$, x_i^T represents the vector of independent variables and β is the regression coefficients. The expected number of events is given by

$$E\left[y_i \,|x_i\,\right] = Var\left[y_i \,|x_i\,\right] = \lambda_i = e^{x_i^T\beta} \tag{3}$$

The log likelihood function is

$$lnL = \sum_{i=1}^{n}\left[-\lambda_i + y_i x_i^T\beta - lny_i!\right] \tag{4}$$

3 Truncated Poisson distribution

Let y a discrete random variable follow Poisson distribution with mean and variance (λ) then the probability mass function of Poisson distribution is [19, 20]:

$$P\left(y_i;\lambda\right) = \frac{e^{-\lambda}\lambda^{y_i}}{y_i!}, \quad y_i = 0, 1, 2, \ldots \quad , \lambda > 0 \tag{5}$$

A sample is considered truncated if the observations are limited to a certain portion of the population distribution. The truncated distribution is a subset of the untruncated distribution, with truncation occurring either from the left side [left truncated ($y > l$)] at a point, l, from the right side [right truncated ($y < k$)] at a point, k, or from both sides within the interval [l, k]. The probability density function of the shortened random variable can be described as a condition a distribution, as demonstrated below [19, 21]:

Case (1): left truncated Poisson at zero ($y_i > 0$)

The zero-truncated Poisson distribution (ZTP) was first introduced by David and Johnson [36], which is distributed as $y \sim ZTP(\lambda)$ and it is one of the models of logarithmic linear regression. The probability mass function of the zero truncated Poisson (ZTP) distribution is [3, 22, 23]:

$$\begin{aligned} P\left(y_i \,|y_i \;> 0\right) &= \frac{P\left(y_i;\lambda\right)}{\Pr\left[y_i > 0\right]} \\ &= \frac{\frac{e^{-\lambda}\lambda^{y_i}}{y_i!}}{1 - \Pr\left[y = 0\right]} \\ &= \frac{e^{-\lambda}\lambda^{y_i}}{(1 - e^{-\lambda})y_i!}, \; y_i = 1, 2, 3, \ldots \end{aligned} \tag{6}$$

Case (2): right truncated Poisson ($y_i \leq k$)

The Poisson distribution becomes right-truncated when truncation occurs at (k) where ($y_i \leq k$). The probability mass function of the right truncated Poisson distribution takes the following form [24, 25]:

$$\begin{aligned} P\left(y_i \,|y_i \leq k\,\right) &= \frac{P\left(y_i;\lambda\right)}{\Pr\left[y_i \leq k\right]} \\ &= \frac{e^{-\lambda}\lambda^{y_i}}{y_i!\left(\sum_{z=0}^{k}\frac{e^{-\lambda}\lambda^z}{z!}\right)} \\ &= \frac{\lambda^{y_i}}{\left(\sum_{z=0}^{k}\frac{\lambda^z}{z!}\right)y_i!}, \; y_i = 0, 1, 2, \ldots, k \end{aligned} \tag{7}$$

Case (3): double truncated Poisson ($l \leq y_i \leq k$)

The Double truncated Poisson data result from merging left truncated and right truncated Poisson data types. The probability mass function of the double truncated Poisson distribution takes the following form [26].

$$\begin{aligned} P\left(y_i | l \leq y_i \leq k\right) &= \frac{P\left(x_i;\lambda\right)}{\Pr\left[l \leq y_i \leq k\right]} \\ &= \frac{\lambda^{y_i}}{\left(\sum_{z=l}^{k}\frac{\lambda^z}{z!}\right)y_i!}, \; y_i = l, l+1, l+2, \ldots, k \end{aligned} \tag{8}$$

The analysis of right truncation and double truncation for count data has received less scholarly focus compared to left truncation. One possible explanation is that Left-truncation occurs more frequently than right-truncation [26].

4 Truncated Poisson regression model

The truncated Poisson regression model is one of the models of logarithmic linear regression for the dependent variable (y_i) and is defined by the following formula [19, 21, 27, 28]:

$$y_i = e^{x_i^T\beta + U_i} \quad U \sim P(\lambda) \tag{9}$$

where $x_i^T\beta = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_n x_{in}$

$$y_i \sim P(\lambda)$$

The distribution parameter of the response variable (y_i) can be expressed as [29–31]:

$$\lambda_i = e^{x_i^T \beta}$$
$$ln\lambda_i = x_i^T \beta$$

In The truncated Poisson regression model the observations of (y_i, x_i) are obtained only for part of the population. The main goal of regression analysis involves parameter estimation to understand the relationship between dependent variable and independent variables. The maximum likelihood estimator serves to calculate parameter estimates for truncated Poisson regression models.

5 Maximum likelihood estimation (MLE)

The estimation of parameters represents a fundamental research topic that attracts mathematical statistics researchers, because new estimation methods require accurate parameter estimation and optimal estimator identification [32].

Case (1): zero truncated Poisson regression model

The maximum likelihood function for the zero truncated Poisson regression model derives from the conditional probability function of the zero truncated Poisson distribution shown in Equation 6.

$$\begin{aligned} L(\beta|y) &= \Pi_{i=1}^{n} \frac{e^{-\lambda}\lambda^{y_i}}{(1-e^{-\lambda})y_i!} \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \ln(\lambda) - \lambda - \ln\left(1-e^{-\lambda}\right) - \ln(y_i!) \right] \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \beta - e^{x_i^T \beta} - \ln\left(1-e^{-x_i^T \beta}\right) - \ln(y_i!) \right] \end{aligned} \tag{10}$$

The zero truncated Poisson maximum likelihood estimators require the derivative of Equation 10 with respect to β to obtain their values as:

$$\frac{\partial lnL}{\partial \beta} = \sum_{i=1}^{n} \left[y_i - \frac{e^{x_i^T \beta}}{(1-e^{-e^{x_i^T \beta}})} \right] x_i = 0 \tag{11}$$

Case (2): right truncated Poisson regression model

The maximum likelihood function for the right truncated Poisson regression model derives from the conditional probability function of the right truncated Poisson distribution shown in Equation 7.

$$\begin{aligned} L(\beta|y) &= \Pi_{i=1}^{n} \frac{\lambda^{y_i}}{\left(\sum_{z=0}^{k} \frac{\lambda^z}{z!}\right) y_i!} \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \ln(\lambda) - \ln(y_i!) - \ln\left(\sum_{z=0}^{k} \frac{\lambda^z}{z!}\right) \right] \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \beta - \ln(y_i!) - \ln\left(\sum_{z=0}^{k} \frac{(x_i^T \beta)^z}{z!}\right) \right] \end{aligned} \tag{12}$$

We derive maximum likelihood right truncated Poisson regression estimators by taking the first derivative of β and setting it to zero according to the following:

$$\frac{\partial lnL}{\partial \beta} = \sum_{i=1}^{n} \left[y_i - \frac{\sum_{z=0}^{k} \frac{x_i z (x_i^T \beta)^z}{z!}}{\sum_{z=0}^{k} \frac{(x_i^T \beta)^z}{z!}} \right] = 0 \tag{13}$$

Case (3): double truncated Poisson regression model

The maximum likelihood function for the double truncated Poisson regression model derives from the Equation 8.

$$\begin{aligned} L(\beta|y) &= \Pi_{i=1}^{n} \frac{\lambda^{y_i}}{\left(\sum_{z=l}^{k} \frac{\lambda^z}{z!}\right) y_i!} \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \ln(\lambda) - \ln(y_i!) - \ln\left(\sum_{z=l}^{k} \frac{\lambda^z}{z!}\right) \right] \\ \ln L(\beta|y) &= \sum_{i=1}^{n} \left[y_i \beta - \ln(y_i!) - \ln\left(\sum_{z=l}^{k} \frac{(x_i^T \beta)^z}{z!}\right) \right] \end{aligned} \tag{14}$$

The maximum likelihood double truncated Poisson regression estimators can be determined differentiating Equation 14 with respect to β, giving

$$\frac{\partial lnL}{\partial \beta} = \sum_{i=1}^{n} \left[y_i - \frac{\sum_{z=l}^{k} \frac{x_i z (x_i^T \beta)^z}{z!}}{\sum_{z=l}^{k} \frac{(x_i^T \beta)^z}{z!}} \right] = 0 \tag{15}$$

Equations 11, 13, 15 contain non-linear relationships between parameters which require iterative methods including Newton Raphson or Fisher scoring. In this research, we employ the traditional optimization method [Quasi newton method (BFGS)] alongside meta heuristic algorithms to estimate parameters of truncated Poisson regression model in three different cases, as well as improving a new algorithm to enhance the solution process.

6 Broyden–Fletcher–Goldfarb–Shanno Method (BFGS)

The BFGS method is one of the quasi-Newton algorithms for unconstrained optimization problem, for finding a point $x^* \in R^n$ let:

$$\min f(x) \quad x^* \in R^n \tag{16}$$

Where the objective function $f : R^n \to R$ is a twice continuously differentiable objective function. The Broyden–Fletcher–Goldfarb–Shanno Method (BFGS) performs an iterative process that follows this procedure [13, 33]:

1. Start with x_1 , an initial point and $[H_1]$, a positive definite symmetric matrix n × n. where $[H_1]$ is the identity matrix $[I]$. Set $i = 1$ the iteration number .

2. Determine the gradient ∇f_i at point x_1, then set:

$$S_i = -[H_1]\nabla f_i \tag{17}$$

3. Determine the optimal step length λ_i^* moving along direction S_i, then set:

$$x_{i+1} = x_i + \lambda_i^* S_i \tag{18}$$

4. Check if the new point x_{i+1} represents an optimal solution. If x_{i+1} is optimal, stop. Otherwise, go to step 5.
5. Update the matrix $[H_1]$ as:

$$\begin{aligned}[H_{i+1}] &= [H_i] + \left(1 + \frac{g_i^T [H_i] g_i}{d_i^T g_i}\right) \\ &\times \frac{d_i d_i^T}{d_i^T g_i} - \frac{d_i g_i^T [H_i]}{d_i^T g_i} - \frac{[H_i]\, g_i d_i^T}{d_i^T g_i}\end{aligned} \tag{19}$$

Where

$$g_i = \nabla f(x_{i+1}) - \nabla f(x_i) = \nabla f_{i+1} - \nabla f_i \tag{20}$$

$$d_i = x_{i+1} - x_i \tag{21}$$

Set $i = i + 1$ the new iteration number, and go to step 2.

7 Black winged kites algorithm (BKA)

In (2024) Wang et al. [15] introduced the black-winged kite optimization algorithm (BKA) which represents a groundbreaking meta-heuristic algorithm. The black-winged kite uses its survival techniques as the basis for its optimization algorithm. This bird utilizes excellent hovering capabilities in addition to its unexpected hunting proficiency. The black-winged kite feeds on insects together with birds and reptiles and small mammals. A model was developed through analysis of black-winged kite movement patterns and hunting abilities [15, 34, 35].

7.1 Initialization

The first step in BKA requires generating random solutions to establish the population as depicted in Algorithm 1. Each Black-winged kite receives its position through a uniform distribution:

$$X_i = BK_{lb} + rand\,(BK_{ub} - BK_{lb}) \tag{22}$$

Where BK_{lb}, BK_{ub} represent the lower and upper bounds of i^{th} black kites, respectively, and $rand \in [0, 1]$ is a random number [15, 34, 35].

7.2 Attacking

The black-winged kite waits quietly before dropping to attack its prey after matching its wings and tail to the wind speed. The black-winged kite underwent two different assault scenarios throughout the global exploration phase of the BKA. The kite maintains its hovering position in the air while it readjusts its position to reach the target at its optimal attack angle. The kite maintains its position in the air while scanning for targets before striking down the most vulnerable one it detects. The attack behavior model uses the following mathematical expression:

$$x_{t+1}^{i,j} = \begin{cases} x_t^{i,j} + n\,(1 + \sin(r)) \times x_t^{i,j} & p < r \\ x_t^{i,j} + n \times (2r - 1) \times x_t^{i,j} & else \end{cases} \tag{23}$$

$$n = 0.05 \times e^{-2*\left(\frac{t}{T}\right)^2} \tag{24}$$

Where $x_{t+1}^{i,j}$, $x_t^{i,j}$ is the position of the i^{th} Black-winged kites in the j^{th} dimension at iteration steps (t) and $(t+1)^{th}$, respectively. $r \in [0, 1]$ is a random number and $p = 0.9$ is a constant value. T represents the total number of iterations and t is the current iteration [15, 34, 35].

7.3 Migration

Bird migration occurs as an intricate behavior because both climate conditions and food availability serve as influential environmental elements. Bird migration exists as an adaptation to seasonal changes through which numerous birds move from northern regions to southern areas for improved living conditions and resources. Migration teams follow leaders who need excellent navigation abilities to achieve success. Our hypothesis relies on bird migration principles which state that if the fitness value of the current population is lower than that of the random population, the leader will give up leadership and join the migratory population, indicating that it is not suitable to lead the population forward. On the other hand, if the fitness value of the current population is higher than that of the random population, then the population will be guided to its destination. The approach enables automatic selection of superior leaders to achieve migration success. The mathematical model describes for the migration patterns of black-winged kites as follows:

$$x_{t+1}^{i,j} = \begin{cases} x_t^{i,j} + C(0,1) \times \left(x_t^{i,j} - L_t^j\right) & F_i < F_{ri} \\ x_t^{i,j} + C(0,1) \times \left(L_t^j - m \times x_t^{i,j}\right) & else \end{cases} \tag{25}$$

$$m = 2 \times \sin(r + \pi/2) \tag{26}$$

The parameter (m) is used to scale the current position of the kite in the update term, and scales the step size toward the leader perturbed by Cauchy mutation.

Where L_t^j is the leading scorer of the black-winged kites in the j^{th} dimension of the t^{th} iteration so far. F_i is the fitness value of the current position obtained by any black-winged

Begin
- Initialization of the position of black-winged kite and evaluation of the objective function
- Calculate the fitness value of each black-winged kite
- **while** (stopping criterion)
- **if** $r < p$, where $r \in [0, 1]$ is a random number
- $x_{t+1}^{i,j} = x_t^{i,j} + n\,(1 + \sin(r)) \times x_t^{i,j}$
- **else**
- $x_{t+1}^{i,j} = x_t^{i,j} + n \times (2r - 1) \times x_t^{i,j}$
- **end if**
- **if** $F_i < F_{ri}$
- $x_{t+1}^{i,j} = x_t^{i,j} + C(0, 1) \times \left(x_t^{i,j} - L_t^j\right)$
- **else**
- $x_{t+1}^{i,j} = x_t^{i,j} + C(0, 1) \times \left(L_t^j - m \times x_t^{i,j}\right)$
- **end if**
- **if** $x_{t+1}^{i,j} < L_t^j$
- $Xbest = x_{t+1}^{i,j}$
- $Fbest = f(x_{t+1}^{i,j})$
- **else** - $Xbest = L_t^j$
- $Fbest = f(L_t^j)$
- **end if**
- **end while**

End

Algorithm 1. Black winged kite algorithm.

kite in the j^{th} dimension of the t^{th} iteration. F_{ri} is the fitness value of the random position obtained from any black kites in the j^{th} dimension of the t^{th} iteration, and $C(0, 1)$ is the Cauchy mutation. The probability density function of the Cauchy distribution is [15, 34, 35]:

$$f(x, \delta, \mu) = \frac{1}{\pi} \frac{\delta}{\delta^2 + (x - \mu)^2} \quad -\infty < x < \infty \tag{27}$$

When $\delta = 0$, $\mu = 1$, then the standard form of the Cauchy distribution becomes the following:

$$f(x, \delta, \mu) = \frac{1}{\pi} \frac{1}{x^2 + 1} \quad -\infty < x < \infty \tag{28}$$

8 Proposed algorithm

In this algorithm, the classical optimization algorithm, BFGS, was combined with the black-winged kite optimization algorithm (BFGS-BKA). where the randomness and speed of the black-winged kite optimization algorithm are used to find the optimal step length λ_i^* in each iteration, while using parameter values before truncated as initial values for BFGS algorithm. The basic steps of this algorithm can be describes as:

Step (1): start with x_1 , an initial point and $[H_1]$, a positive definite symmetric matrix n × n. where $[H_1]$ is the identity matrix $[I]$. Set $i = 1$ the iteration number.

Step (2): determine the gradient ∇f_i at point x_1 , then set:

$$S_i = -[H_1]\,\nabla f_i$$

TABLE 1 Results zero-truncated Poisson regression model for the medpar data.

Parameters	Methods			
	Newton Raphson	DFP	BFGS	BFGS-BKA
Constant	2.33286	2.33286	2.33286	2.33281
White	−0.1539437	−0.1539437	−0.1539437	−0.1539429
Hmo	−0.0716485	−0.0716485	−0.0716485	−0.0716484
Type2	0.2217806	0.2217807	0.2217806	0.2217802
Type3	0.7096162	0.7096164	0.7096162	0.7096157
MSE	0.0225	0.0225	0.0225	0.0211

Step (3): find the optimal step length λ_i^* by black-winged kite optimization algorithm:

1. Randomly generate the initial population of black-winged kite between λ_{lb} and λ_{ub} as λ_1, $\lambda_2, \ldots\ldots, \lambda_N$. Evaluate the fitness value of each black-winged kite as (λ_1) , $f(\lambda_2)$, ..., $f(\lambda_N)$. Set $t = 1$ the iteration number.
2. BKA chooses the individual with the best fitness value to become the leader λ_L in the initial population.
3. Find f_{best} and λ_L , the black-winged kite algorithm initiates its global exploration and search during its attack behavior according the following equation:

$$\lambda_{t+1}^{i,j} = \begin{cases} \lambda_t^{i,j} + n\,(1 + \sin(r)) \times \lambda_t^{i,j} & p < r \\ \lambda_t^{i,j} + n \times (2r - 1) \times \lambda_t^{i,j} & else \end{cases}$$

4. In Bird migration , will be the position is update based on the fitness value of the leader as:

$$\lambda_{t+1}^{i,j} = \begin{cases} \lambda_t^{i,j} + C(0, 1) \times \left(\lambda_t^{i,j} - L_t^j\right) & F_i < F_{ri} \\ \lambda_t^{i,j} + C(0, 1) \times \left(L_t^j - m \times \lambda_t^{i,j}\right) & else \end{cases}$$

5. Test the convergence of the current solution. If the convergence criterion is not satisfied, go to step (3) and update the iteration number as $t = t + 1$, until convergence occurs and the optimal value of λ is determined.

Step (4): find new point:

$$x_{i+1} = x_i + \lambda_i^* S_i$$

Step (5): check if the new point x_{i+1} represents an optimal solution. If x_{i+1} is optimal, stop. Otherwise, go to step 6.

Step (6): update the matrix $[H_1]$ as:

$$[H_{i+1}] = [H_i] + \left(1 + \frac{g_i^T [H_i]\, g_i}{d_i^T g_i}\right) \frac{d_i d_i^T}{d_i^T g_i} - \frac{d_i g_i^T [H_i]}{d_i^T g_i} - \frac{[H_i]\, g_i d_i^T}{d_i^T g_i}$$

Where

$$g_i = \nabla f(x_{i+1}) - \nabla f(x_i) = \nabla f_{i+1} - \nabla f_i$$
$$d_i = x_{i+1} - x_i$$

Step (7): set $i = i+1$ the new iteration number, and go to step 2.

10.3389/fams.2026.1744058

TABLE 2 Simulation results for estimating parameters of a left truncated Poisson regression model at zero ($y > 0$; P = 3).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	0.0605861	0.0606416	0.0606268	0.0606055
	β_1	0.9904165	0.9904273	0.9904202	0.9904173
	β_2	−0.524277	−0.5242166	−0.5242486	−0.5242437
	β_3	0.009349	0.0093336	0.0093399	0.0093384
	MSE	0.1822	0.1804	0.1801	**0.1786**
50	β_0	−0.2315287	−0.2315281	−0.2315132	−0.2315116
	β_1	0.7735502	0.7735496	0.7735485	0.7735473
	β_2	−0.3058557	−0.3058522	−0.3058719	−0.3058719
	β_3	0.3230417	0.3230426	0.3230253	0.3230234
	MSE	0.1477	0.1459	0.1453	**0.1418**
100	β_0	0.0433403	0.0433412	0.0433433	0.0433433
	β_1	0.5843068	0.5843067	0.5843021	0.5843008
	β_2	−0.4143445	−0.4143451	−0.4143427	−0.4143419
	β_3	0.0748848	0.0748851	0.0748824	0.0748806
	MSE	0.0952	0.0947	0.0938	**0.0915**
200	β_0	−0.0092981	−0.009296	−0.0092982	−0.0092982
	β_1	0.7450347	0.7450345	0.7450346	0.7450338
	β_2	−0.4905174	−0.4905167	−0.4905176	−0.4905176
	β_3	0.2223349	0.2223347	0.222335	0.2223267
	MSE	0.0773	0.0769	0.0767	**0.0745**

The best results are in bold.

9 Real data

This section contains real dataset to demonstrate the empirical importance of the algorithm (BFGS-BKA), where the real dataset is for the zero truncated Poisson regression model study. We use the (Arizona MedPar database, 1991). The dataset contains 1,495 observations the response variable, Lose, represents length of hospital stay. The explanatory variables for this model include an indicator of White (Patient identifies themselves as Caucasian, binary), Hmo (Patient belongs to a Health Maintenance Organization, binary), Type2 (Urgent admission, binary) and Type3 (Elective admission, binary).

As illustrated by Table 1, across all four methods, the parameter estimates are remarkably consistent, indicating stable estimation despite the choice of optimization algorithm. The numerical differences between the methods on each parameter are very small showing that all methods converge to nearly identical solutions. The MSE measures the average squared difference between observed and predicted values, serving as an indicator of model fit quality and estimator accuracy. Newton Raphson, Davidon–Fletcher–Powell (DFP) method, and BFGS yield the same MSE value of 0.0225, whereas the BFGS-BKA method achieves a slightly lower MSE of 0.0211. This implies that integrating the BKA algorithm with BFGS potentially enhances estimation accuracy or predictive performance. This improvement indicates that BFGS-BKA may better navigate the complex truncated likelihood surface to find a more optimal parameter set or avoid local optima.

10 Simulation study

This section presented a simulation study to demonstrate the empirical importance of the algorithm (BFGS-BKA) to evaluate the accuracy of MLEs of parameters estimation of a truncated Poisson regression model of three cases left truncated Poisson regression model (LTPRM), right truncated Poisson regression model (RTPRM) and double truncated Poisson regression model (DTPRM). We consider dimension $P = 3, 7, 12$ and the sample sizes $n = 25, 50, 100, 200$.

10.1 Simulation results of the left truncated Poisson regression model

This section represented results for fit the left truncated Poisson regression model, with responses truncated at L = 0. Where independent variables are randomly generated according to a normal distribution and the data were simulated with true values beta = [0.5; −0.3; 0.2], beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2] and beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2; 0.4; −0.25; 0.15; 0.05; −0.05]. Tables 2–4 show the simulation results for LTPRM.

TABLE 3 Simulation results for estimating parameters of a left truncated Poisson regression model at zero ($y > 0$; P = 7).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	0.0926978	0.0926949	0.0927003	0.0927015
	β_1	0.564459	0.564461	0.5644587	0.5644587
	β_2	−0.6440744	−0.6440738	−0.6440724	−0.6440724
	β_3	0.280506	0.2805056	0.2805057	0.2805057
	β_4	0.069465	0.069467	0.0694647	0.0694631
	β_5	0.1659867	0.1659801	0.1659896	0.1659896
	β_6	0.22054	0.2205372	0.2205399	0.2205399
	β_7	0.1552263	0.155215	0.1552279	0.1552279
	MSE	0.2515	0.2509	0.2507	**0.2484**
50	β_0	−0.3173069	−0.3173081	−0.3173909	−0.3173909
	β_1	0.655645	0.6556289	0.6557447	0.6557432
	β_2	−0.2523863	−0.252384	−0.2523919	−0.25239
	β_3	0.3195793	0.3195883	0.3195612	0.3195612
	β_4	0.1808037	0.1808076	0.1808408	0.1808403
	β_5	−0.0684644	−0.0684646	−0.0685744	−0.0685744
	β_6	0.3032056	0.303196	0.303276	0.3032741
	β_7	−0.2861774	−0.286179	−0.2862013	−0.2862007
	MSE	0.2219	0.2216	0.2215	**0.2201**
100	β_0	−0.1071277	−0.1071258	−0.1071276	−0.107126
	β_1	0.7374492	0.7374496	0.7374505	0.7374505
	β_2	−0.1614437	−0.161447	−0.1614488	−0.1614488
	β_3	0.3848424	0.3848399	0.3848526	0.3848512
	β_4	0.01762	0.0176189	0.0176141	0.0176123
	β_5	0.0574704	0.057468	0.0574725	0.0574725
	β_6	0.3110027	0.3110044	0.3110038	0.3110038
	β_7	−0.4387681	−0.438765	−0.4387697	−0.4387697
	MSE	0.1042	0.1040	0.1039	**0.1021**
200	β_0	−0.2050698	−0.2050574	−0.2050697	−0.2050696
	β_1	0.5836933	0.5836848	0.5836637	0.5836614
	β_2	−0.2069601	−0.2069578	−0.2069618	−0.2069618
	β_3	0.3581892	0.3581868	0.3581863	0.3581846
	β_4	0.0030445	0.0030671	0.0030308	0.0030302
	β_5	−0.202504	−0.2024983	−0.2025151	−0.2025151
	β_6	0.4400861	0.440103	0.4400701	0.4400701
	β_7	−0.2052846	−0.2052629	−0.2052789	−0.2052763
	MSE	0.0945	0.0943	0.0941	**0.0915**

The best results are in bold.

10.2 Simulation results of the right truncated Poisson regression model

This section represented results for fit the right truncated Poisson regression model, with responses truncated at U = 5. Where independent variables are randomly generated according to a normal distribution using matlab randn (n,p), also the data were simulated with true values beta = [0.5; −0.3; 0.2] , beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2] and beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2; 0.4; −0.25; 0.15; 0.05; −0.05]. Tables 5–7 show the simulation results for (RTPRM).

TABLE 4 Simulation results for estimating parameters of a left truncated Poisson regression model at zero ($y > 0$; P = 12).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.8614353	−0.8614325	−0.861444	−0.861427
	β_1	0.7035994	0.7035977	0.7036011	0.7036014
	β_2	−0.1529654	−0.1529619	−0.1529639	−0.1529633
	β_3	0.48303	0.4830395	0.4830308	0.4830301
	β_4	−0.7203112	−0.7203052	−0.7203215	−0.7203212
	β_5	0.3923419	0.3923434	0.3923467	0.3923471
	β_6	0.0631014	0.0631054	0.0630985	0.0630964
	β_7	−0.0710695	−0.0710814	−0.0710717	−0.071072
	β_8	−0.8822538	−0.8822546	−0.8822576	−0.8822573
	β_9	0.0181201	0.0181137	0.0181281	0.018128
	β_{10}	−0.6478624	−0.6478539	−0.6478666	−0.6478666
	β_{11}	0.4385116	0.4385079	0.4385171	0.4385169
	β_{12}	−0.3494832	−0.3494796	−0.3494841	−0.3494845
	MSE	0.3152	0.3149	0.3147	**0.3124**
50	β_0	0.2450007	0.2450028	0.2450011	0.2450013
	β_1	0.5359116	0.5359177	0.5359103	0.5359106
	β_2	−0.4534861	−0.4534876	−0.4534873	−0.4534871
	β_3	0.1330119	0.1330141	0.1330123	0.1330123
	β_4	0.1638947	0.1638883	0.1638955	0.1638962
	β_5	−0.0805194	−0.0805205	−0.0805195	−0.0805197
	β_6	0.4945398	0.494542	0.4945397	0.4945393
	β_7	−0.1883591	−0.1883612	−0.1883582	−0.1883579
	β_8	0.4052023	0.4052011	0.4052019	0.4052012
	β_9	−0.059146	−0.0591436	−0.0591467	−0.0591471
	β_{10}	−0.0556521	−0.0556525	−0.0556508	−0.0556504
	β_{11}	−0.12719	−0.127192	−0.1271926	−0.1271935
	β_{12}	0.0566836	0.0566845	0.056682	0.0566878
	MSE	0.2147	0.2144	0.2139	**0.2112**
100	β_0	−0.118253	−0.1182646	−0.1182716	−0.1182834
	β_1	0.6130675	0.6130611	0.6130748	0.6130739
	β_2	−0.3620159	−0.3620285	−0.3620218	−0.3620213
	β_3	0.0938253	0.0938262	0.0938282	0.0938291
	β_4	0.1551607	0.1551616	0.1551665	0.1551687
	β_5	−0.1028906	−0.102884	−0.1028899	−0.1028899
	β_6	0.3558426	0.3558493	0.3558494	0.3558498
	β_7	−0.0969868	−0.0969838	−0.0969893	−0.0969895
	β_8	0.3841613	0.3841709	0.3841673	0.3841672
	β_9	−0.1864775	−0.1864835	−0.1864828	−0.1864822
	β_{10}	0.2535725	0.253565	0.2535692	0.2535697
	β_{11}	−0.2586034	−0.2586041	−0.2586022	−0.2586019
	β_{12}	−0.0180681	−0.0180704	−0.0180699	−0.0180699
	MSE	0.1132	0.1128	0.1126	**0.1113**

(Continued)

TABLE 4 (Continued)

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
200	β_0	−0.2619105	−0.2619025	−0.2618937	−0.2618923
	β_1	0.5147301	0.5147283	0.5147267	0.5147252
	β_2	−0.3444794	−0.3444768	−0.3444744	−0.3444749
	β_3	0.1516148	0.1516168	0.1516111	0.1516112
	β_4	0.0727301	0.0727304	0.0727338	0.0727341
	β_5	−0.0577983	−0.0577959	−0.0577988	−0.0577988
	β_6	0.4405197	0.4405187	0.4405171	0.4405179
	β_7	−0.335039	−0.3350374	−0.3350321	−0.3350314
	β_8	0.4311549	0.4311539	0.4311521	0.4311528
	β_9	−0.1879824	−0.1879845	−0.1879841	−0.187984
	β_{10}	0.1386025	0.1386018	0.1386025	0.1386025
	β_{11}	0.0912978	0.0912964	0.0912963	0.0912961
	β_{12}	−0.05895	−0.0589494	−0.0589534	−0.0589531
	MSE	0.0823	0.0821	0.0818	**0.0802**

The best results are in bold.

TABLE 5 Simulation results for estimating parameters of a right truncated Poisson regression model (P = 3).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.2044458	−0.2047354	−0.2045128	−0.2045012
	β_1	0.7133822	0.7136196	0.713473	0.7132561
	β_2	−0.6439646	−0.64431	−0.6440023	−0.6437815
	β_3	0.0267832	0.0265996	0.0266998	0.0266897
	MSE	0.1829	0.1802	0.1801	**0.1789**
50	β_0	0.1481886	0.1481877	0.1481885	0.1481875
	β_1	0.4981213	0.498122	0.4981215	0.4981203
	β_2	−0.5420273	−0.5420278	−0.5420273	−0.5420264
	β_3	0.3070644	0.3070648	0.3070653	0.3070621
	MSE	0.1946	0.1931	0.1911	**0.1909**
100	β_0	−0.0986433	−0.0986435	−0.0986435	−0.0986435
	β_1	0.5020687	0.5020689	0.5020689	0.5020689
	β_2	−0.338183	−0.338183	−0.3381831	−0.3381831
	β_3	0.1941197	0.1941197	0.1941197	0.1941197
	MSE	0.0826	0.0813	0.0806	**0.0801**
200	β_0	0.0338927	0.0338926	0.0338927	0.033893
	β_1	0.5449919	0.5449919	0.5449919	0.5449919
	β_2	−0.2905865	−0.2905865	−0.2905865	−0.2905865
	β_3	0.2194149	0.2194149	0.2194149	0.2194149
	MSE	0.0716	0.0715	0.0714	**0.0701**

The best results are in bold.

TABLE 6 Simulation results for estimating parameters of a right truncated Poisson regression model (P = 7).

Sample size (*n*)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.4794763	−0.4794751	−0.4794751	−0.4794751
	β_1	0.5107009	0.5107017	0.5107012	0.5107006
	β_2	0.1826785	0.1826817	0.1826795	0.1826711
	β_3	0.3178249	0.3178251	0.3178251	0.3178251
	β_4	0.4424957	0.4424962	0.4424953	0.442494
	β_5	−0.3870675	−0.3870688	−0.3870697	−0.3870697
	β_6	0.8706261	0.8706264	0.8706269	0.8706269
	β_7	0.1193511	0.1193497	0.1193495	0.119349
	MSE	0.2547	0.2545	0.2543	**0.2532**
50	β_0	−0.047451	−0.0474496	−0.0474511	−0.0474511
	β_1	0.0796038	0.0796035	0.079604	0.079604
	β_2	−0.2130617	−0.2130617	−0.2130618	−0.2130618
	β_3	−0.1061506	−0.1061513	−0.1061506	−0.1061502
	β_4	0.6912239	0.6912238	0.691224	0.6912242
	β_5	−0.2849013	−0.2849013	−0.2849013	−0.2849013
	β_6	0.0200921	0.0200916	0.0200923	0.0200925
	β_7	−0.0268927	−0.0268926	−0.0268927	−0.0268927
	MSE	0.2031	0.2027	0.2025	**0.2012**
100	β_0	−0.0201553	−0.0201562	−0.0201553	−0.0201458
	β_1	0.3946759	0.3946762	0.3946759	0.3946742
	β_2	−0.1337339	−0.1337337	−0.1337339	−0.1337339
	β_3	0.0826245	0.0826248	0.0826245	0.0826245
	β_4	0.4280978	0.4280989	0.4280978	0.4280978
	β_5	−0.1179131	−0.1179137	−0.1179131	−0.1179131
	β_6	0.3534397	0.3534403	0.3534397	0.3534397
	β_7	−0.0139108	−0.0139105	−0.0139108	−0.0139108
	MSE	0.1132	0.1131	0.1129	**0.1118**
200	β_0	0.003164	0.003164	0.003164	0.003164
	β_1	0.3648677	0.3648678	0.3648677	0.3648669
	β_2	−0.1732149	−0.1732149	−0.1732149	−0.1732149
	β_3	0.0008671	0.0008671	0.0008671	0.0008671
	β_4	0.5622831	0.5622832	0.5622832	0.5622825
	β_5	−0.0834699	−0.0834699	−0.0834698	−0.0834696
	β_6	0.3784646	0.3784646	0.3784646	0.3784646
	β_7	0.1480245	0.1480245	0.1480245	0.1480245
	MSE	0.0831	0.0827	0.0824	**0.0813**

The best results are in bold.

10.3 Simulation results of the double truncated Poisson regression model

This section represented results for fit the double truncated Poisson regression model, with responses truncated at L = 0 and U = 5. Where Independent variables are randomly generated according to a normal distribution using matlab randn (n,p), also the data were simulated with true values beta = [0.5; −0.3; 0.2] , beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2] and beta = [0.5; −0.3; 0.2; 0.1; −0.1; 0.3; −0.2; 0.4; −0.25;

TABLE 7 Simulation results for estimating parameters of a right truncated Poisson regression model (P = 12).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	0.5074047	0.5074047	0.5074047	0.5074047
	β_1	0.2848892	0.2848892	0.2848892	0.2848892
	β_2	−0.4693933	−0.4693933	−0.4693933	−0.4693933
	β_3	0.5707239	0.5707239	0.5707239	0.5707239
	β_4	0.0818152	0.0818152	0.0818152	0.0818152
	β_5	−0.1785226	−0.1785226	−0.1785226	−0.1785223
	β_6	0.9287227	0.9287227	0.9287227	0.9287227
	β_7	−0.4038411	−0.4038411	−0.4038411	−0.4038411
	β_8	0.3582214	0.3582214	0.3582214	0.3582214
	β_9	−0.1126234	−0.1126234	−0.1126234	−0.1126227
	β_{10}	0.5869459	0.5869459	0.5869459	0.5869459
	β_{11}	0.0692677	0.0692677	0.0692677	0.0692677
	β_{12}	−0.0358698	−0.0358698	−0.0358698	−0.0358698
	MSE	0.3163	0.3161	0.3160	**0.3141**
50	β_0	−0.4827961	−0.4828029	−0.4828315	−0.4828273
	β_1	0.4601612	0.460162	0.4601896	0.4601899
	β_2	−0.6072522	−0.6072504	−0.6072636	−0.6072651
	β_3	0.069667	0.0696696	0.0696657	0.0696643
	β_4	0.1243769	0.1243796	0.1244038	0.1244038
	β_5	−0.4199463	−0.419951	−0.4200035	−0.4200035
	β_6	0.3623213	0.3623205	0.3623135	0.3623116
	β_7	−0.2892603	−0.2892586	−0.2892488	−0.2892465
	β_8	0.9695849	0.9695876	0.9696129	0.9696129
	β_9	−0.0936884	−0.0936863	−0.0936845	−0.0936837
	β_{10}	0.612861	0.6128638	0.612854	0.612854
	β_{11}	0.1164395	0.1164401	0.1164395	0.1164395
	β_{12}	0.1428553	0.1428585	0.1428394	0.14283622
	MSE	0.2338	0.2335	0.2332	**0.2217**
100	β_0	−0.0069453	−0.0069457	−0.0069454	−0.0069454
	β_1	0.4531302	0.4531303	0.4531302	0.4531291
	β_2	−0.335599	−0.335599	−0.335599	−0.335599
	β_3	0.1139658	0.1139659	0.1139658	0.1139648
	β_4	0.0987151	0.0987151	0.0987151	0.0987151
	β_5	−0.1070247	−0.1070247	−0.1070247	−0.1070247
	β_6	0.1939749	0.1939748	0.1939749	0.1939749
	β_7	−0.3772053	−0.3772055	−0.3772054	−0.3772054
	β_8	0.491455	0.4914553	0.4914551	0.4914551
	β_9	−0.4061271	−0.4061273	−0.4061272	−0.406127
	β_{10}	0.0180528	0.018053	0.0180528	0.0180519
	β_{11}	0.0238667	0.0238668	0.0238668	0.0238668
	β_{12}	−0.2245255	−0.2245256	−0.2245255	−0.2245255
	MSE	0.1255	0.1253	0.1251	**0.1232**

(Continued)

TABLE 7 (Continued)

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
200	β_0	−0.0483217	−0.0483214	−0.048323	−0.048323
	β_1	0.5104306	0.5104302	0.5104312	0.5104322
	β_2	−0.3227727	−0.3227724	−0.3227734	−0.3227734
	β_3	0.0754467	0.0754466	0.0754464	0.0754458
	β_4	0.0824297	0.0824298	0.08243	0.0824312
	β_5	−0.1066715	−0.1066716	−0.1066719	−0.106672
	β_6	0.2741968	0.2741964	0.2741966	0.2741965
	β_7	−0.2391153	−0.2391153	−0.2391158	−0.2391158
	β_8	0.49603	0.4960291	0.4960302	0.4960302
	β_9	−0.2210014	−0.2210012	−0.221002	−0.2210026
	β_{10}	0.2310687	0.2310681	0.2310689	0.2310689
	β_{11}	0.0333784	0.0333783	0.0333785	0.0333774
	β_{12}	−0.0125663	−0.0125661	−0.0125661	−0.0125661
	MSE	0.0945	0.0942	0.0939	**0.0921**

The best results are in bold.

TABLE 8 Simulation results for estimating parameters of a double truncated Poisson regression model (P = 3).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.1558273	−0.1557939	−0.1557566	−0.1557506
	β_1	0.4512201	0.4511945	0.4511494	0.4511398
	β_2	0.1766003	0.1766157	0.17659	0.1765241
	β_3	0.4351908	0.4351774	0.4351864	0.4351864
	MSE	0.2955	0.2821	0.2763	**0.2551**
50	β_0	−0.2589921	−0.2589981	−0.2590017	−0.2590017
	β_1	0.4438434	0.4438345	0.4438293	0.4438229
	β_2	0.4488448	0.4488376	0.4488351	0.4488305
	β_3	0.5731984	0.5731992	0.573202	0.573202
	MSE	0.2331	0.2286	0.2147	**0.2131**
100	β_0	−0.1156143	−0.115618	−0.1156356	−0.1156356
	β_1	0.3477845	0.3477863	0.3477778	0.3477746
	β_2	0.3157024	0.3157139	0.3157184	0.3157184
	β_3	0.2233149	0.2233078	0.2233294	0.2233267
	MSE	0.1621	0.1618	0.1605	**0.1587**
200	β_0	−0.0226799	−0.022708	−0.0226883	−0.0226848
	β_1	0.517915	0.5179045	0.5179474	0.5179462
	β_2	0.3185699	0.3185493	0.3185984	0.3185983
	β_3	0.4484079	0.4483947	0.4484859	0.4484837
	MSE	0.1159	0.1077	0.1054	**0.1018**

The best results are in bold.

TABLE 9 Simulation results for estimating parameters of a double truncated Poisson regression model (P = 7).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.2858235	−0.2857848	−0.2858225	−0.2858223
	β_1	−0.1653242	−0.1653189	−0.1653308	−0.1653312
	β_2	0.2407589	0.2407697	0.240767	0.2407665
	β_3	0.8443157	0.8442951	0.8442992	0.8442997
	β_4	0.3714291	0.3714186	0.3714336	0.3714342
	β_5	−0.2360293	−0.2360385	−0.236036	−0.2360356
	β_6	0.3906935	0.3906461	0.3906882	0.3906893
	β_7	−0.5665927	−0.566663	−0.5665713	−0.5665721
	MSE	0.3215	0.3208	0.3195	**0.3174**
50	β_0	−0.0375944	−0.0375962	−0.0376234	−0.0376282
	β_1	0.66365	0.6636705	0.6636389	0.663639
	β_2	−0.1675661	−0.1675437	−0.1675875	−0.1675894
	β_3	0.345409	0.3453086	0.3454378	0.3454386
	β_4	−0.0118677	−0.0118937	−0.0117899	−0.011789
	β_5	0.0154284	0.0154216	0.0154415	0.0154432
	β_6	0.258271	0.2583602	0.2583117	0.2583148
	β_7	−0.3189341	−0.3189986	−0.3189528	−0.3189539
	MSE	0.3081	0.3025	0.3018	**0.3004**
100	β_0	−0.2282489	−0.2282489	−0.228258	−0.228242
	β_1	0.3241313	0.3241313	0.3241238	0.3241238
	β_2	−0.1384214	−0.1384214	−0.13842	−0.138367
	β_3	0.5801789	0.5801789	0.5801797	0.5801854
	β_4	0.0858746	0.0858746	0.0858833	0.0858864
	β_5	−0.0348083	−0.0348083	−0.0348339	−0.0348425
	β_6	0.5331769	0.5331769	0.533192	0.5331974
	β_7	−0.2537133	−0.2537133	−0.2537146	−0.2537185
	MSE	0.2137	0.2136	0.2041	**0.2028**
200	β_0	−0.1587123	−0.1587184	−0.1587097	−0.1587068
	β_1	0.6670115	0.6670268	0.667012	0.6670125
	β_2	−0.5487419	−0.5487332	−0.5487403	−0.5487401
	β_3	0.3911965	0.3911697	0.3911951	0.3911955
	β_4	−0.0013219	−0.0013251	−0.0013203	−0.00132
	β_5	−0.0321376	−0.0321579	−0.0321368	−0.0321347
	β_6	0.4452084	0.4451819	0.4452131	0.4452159
	β_7	−0.1397936	−0.1398144	−0.1397963	−0.1397963
	MSE	0.1486	0.1337	0.1314	**0.1289**

The best results are in bold.

0.15; 0.05; −0.05]. Tables 8–10 show the simulation results for (DTPRM).

From all the Tables above, we can concluded that across all sample sizes, parameter estimates from all four methods are highly similar, demonstrating consistent convergence to approximately the same values. This consistency suggests each optimization method is capable of locating reliable parameter estimates even in smaller samples. As sample size increases, the parameter estimates stabilize and vary less across methods. This reflects the expected property of maximum likelihood estimators: increased sample size

TABLE 10 Simulation results for estimating parameters of a double truncated Poisson regression model (P = 12).

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
25	β_0	−0.6073089	−0.6073912	−0.607189	−0.607152
	β_1	−0.1323503	−0.1323735	−0.1323581	−0.1323575
	β_2	0.3691619	0.3691301	0.3690593	0.3690581
	β_3	−0.259072	−0.2590905	−0.259066	−0.2590571
	β_4	−0.1889251	−0.1889383	−0.1888898	−0.1888643
	β_5	−0.5384405	−0.5384847	−0.538497	−0.538499
	β_6	0.6892821	0.6893039	0.6893448	0.6893673
	β_7	−0.3449217	−0.3449529	−0.344885	−0.344839
	β_8	−0.2177979	−0.217815	−0.2178416	−0.2178625
	β_9	−0.5734173	−0.5734628	−0.5733326	−0.5733308
	β_{10}	−0.3854379	−0.3854759	−0.385406	−0.385392
	β_{11}	1.314073	1.314133	1.313973	1.313947
	β_{12}	1.279418	1.279486	1.279205	1.279201
	MSE	0.4132	0.4085	0.4055	**0.4031**
50	β_0	0.0260518	0.0259643	0.0260443	0.0260427
	β_1	0.2475601	0.2475584	0.2475476	0.2475293
	β_2	−0.2577356	−0.2577761	−0.2577354	−0.2577316
	β_3	0.0468957	0.0469244	0.0468872	0.0468621
	β_4	−0.1204874	−0.1204298	−0.1204943	−0.1204997
	β_5	−0.4188021	−0.4188395	−0.4187998	−0.4187205
	β_6	0.047606	0.0475921	0.0476052	0.0476044
	β_7	−0.0003571	−0.0002965	−0.0003472	−0.0003472
	β_8	0.1689888	0.1689627	0.1689886	0.1689881
	β_9	−0.9068353	−0.9069006	−0.9068387	−0.9068392
	β_{10}	0.178105	0.178127	0.1781101	0.1781113
	β_{11}	0.4888108	0.4888359	0.4888204	0.4888252
	β_{12}	0.3629748	0.3630487	0.3629835	0.3629927
	MSE	0.3462	0.3255	0.3032	**0.3001**
100	β_0	0.0385659	0.0385516	0.0385577	0.0385582
	β_1	0.4102414	0.410243	0.4102418	0.4102437
	β_2	−0.6420943	−0.642092	−0.6421023	−0.6421035
	β_3	0.0022714	0.0022528	0.0022711	0.0022742
	β_4	−0.2230076	−0.2230055	−0.2230088	−0.2230089
	β_5	−0.1790879	−0.1790891	−0.1790935	−0.1790964
	β_6	0.355542	0.3555414	0.3555464	0.3555483
	β_7	−0.1333147	−0.133325	−0.1333074	−0.1333055
	β_8	0.4505356	0.4505381	0.4505453	0.4505461
	β_9	−0.4601778	−0.460187	−0.4601826	−0.4601815
	β_{10}	0.4124141	0.4124227	0.4124199	0.4124176
	β_{11}	0.1994184	0.1994095	0.1994132	0.1994264
	β_{12}	−0.1919182	−0.1919244	−0.1919197	−0.1919199
	MSE	0.2046	0.2039	0.2008	**0.1991**

(Continued)

TABLE 10 (Continued)

Sample size (n)	Parameters	Methods			
		Newton Raphson	DFP	BFGS	BFGS-BKA
200	β_0	−0.1014937	−0.1014946	−0.1015082	−0.1015153
	β_1	0.4214451	0.4214462	0.4214523	0.4214592
	β_2	−0.3636477	−0.3636489	−0.3636515	−0.3636594
	β_3	−0.2367109	−0.2367112	−0.2367138	−0.2367182
	β_4	−0.1965457	−0.1965465	−0.19655	−0.1965528
	β_5	−0.2376769	−0.2376772	−0.2376771	−0.2376771
	β_6	0.2626723	0.2626748	0.2626743	0.262674
	β_7	−0.1748725	−0.1748717	−0.1748697	−0.1748641
	β_8	0.5248844	0.5248844	0.5248921	0.5248938
	β_9	−0.3120583	−0.3120575	−0.3120569	−0.3120554
	β_{10}	0.1084512	0.108451	0.1084503	0.1084489
	β_{11}	0.0542112	0.0542105	0.0542091	0.0542063
	β_{12}	−0.0990921	−0.0990901	−0.0990931	−0.0990942
	MSE	0.1387	0.1255	0.1208	**0.1135**

The best results are in bold.

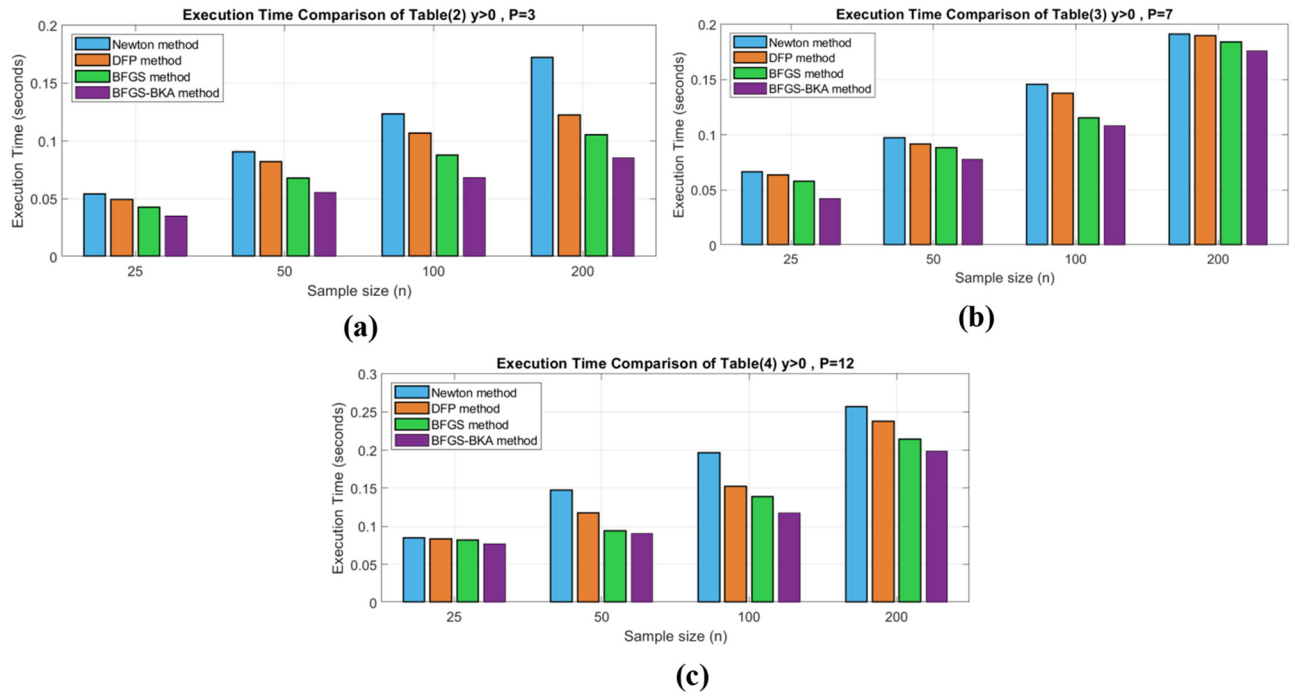

FIGURE 1
Computational time of left truncated Poisson regression model at zero (y > 0) **(a)** P = 3, **(b)** P = 7 and **(c)** P = 12.

yields more precise and stable estimates. As sample size increases (from 25 to 200), the estimated MSE decreases monotonically for all methods, demonstrating improved estimation precision with larger data consistent with statistical theory. BFGS-BKA consistently achieves the lowest MSE at each sample size, indicating superior estimation accuracy. Classical methods such as Newton-Raphson, DFP, and BFGS provide similar MSE values, slightly higher than BFGS-BKA. The performance gap, though sometimes small in absolute terms, illustrates the advantage of using the Black Kite Optimization (BKA) to enhance the classical BFGS method, improving optimization over the complex truncated likelihood surface. The similarity of parameter estimates across methods confirms robustness of the numerical algorithms when applied to truncated Poisson regression models.

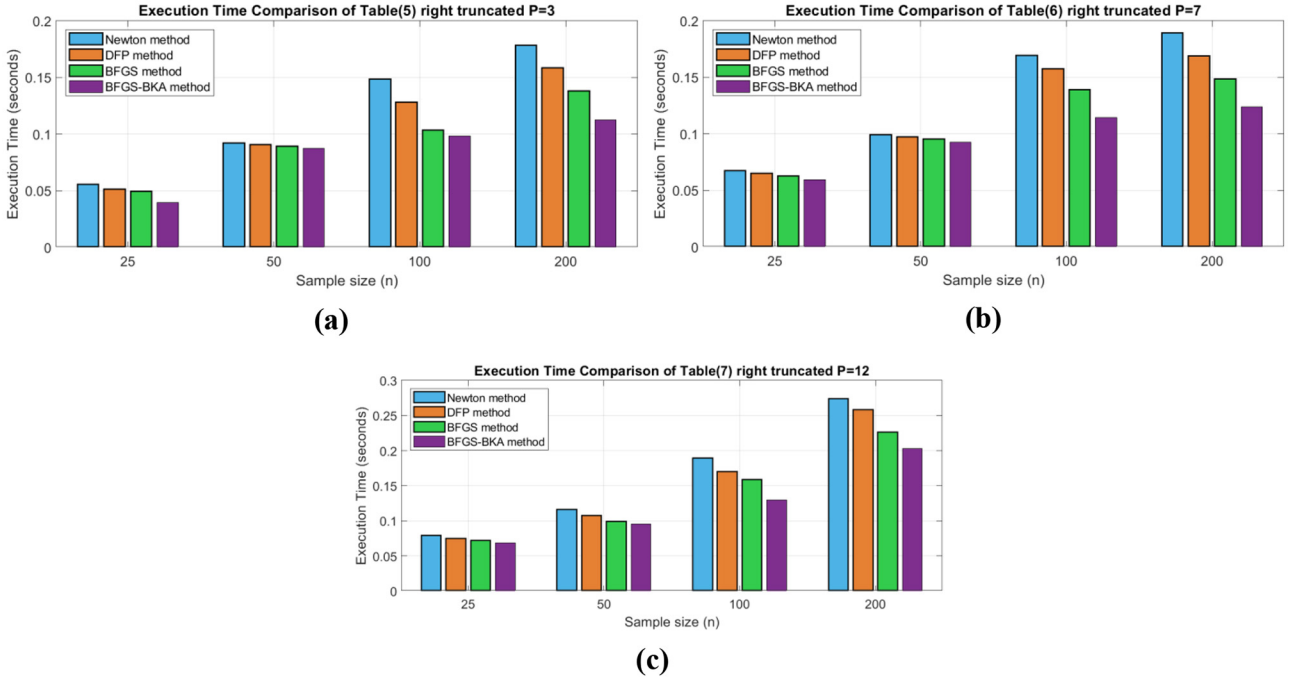

FIGURE 2
Computational time of right truncated Poisson regression model **(a)** P = 3, **(b)** P = 7 and **(c)** P = 12.

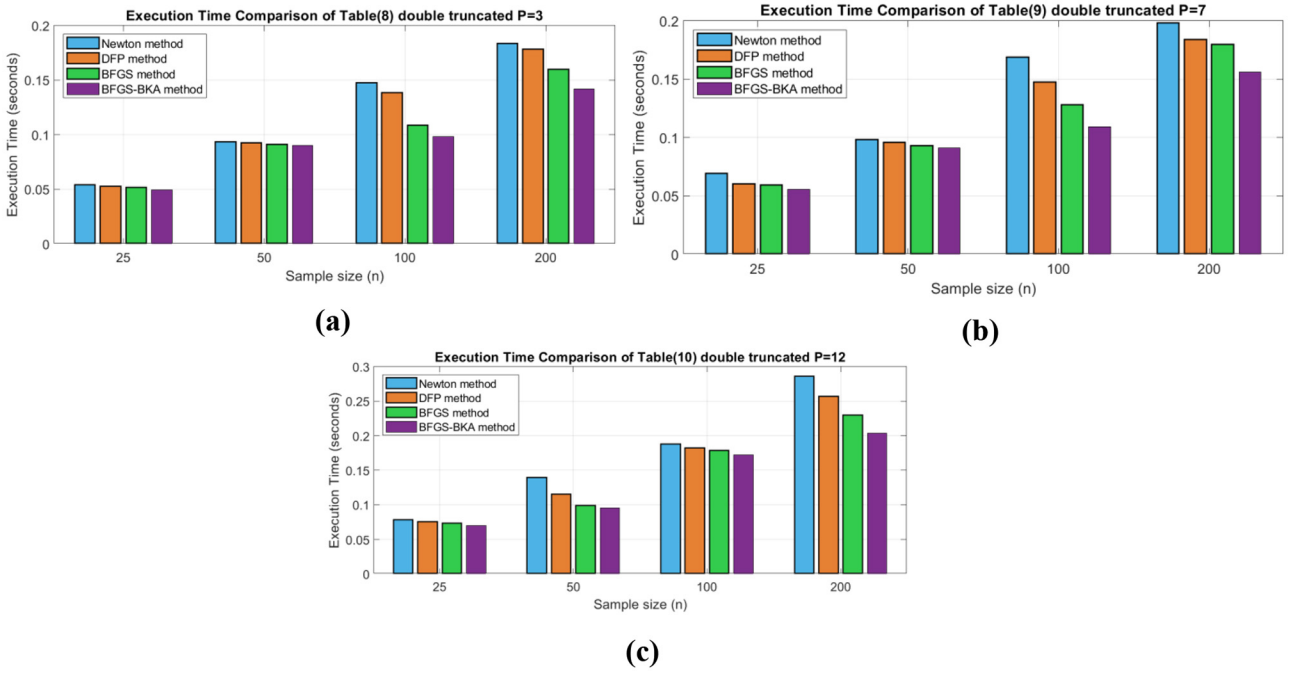

FIGURE 3
Computational time of double truncated Poisson regression model **(a)** P = 3, **(b)** P = 7 and **(c)** P = 12.

Integrating the Black Kite Optimization algorithm with BFGS consistently improves the optimization process, yielding more accurate and stable estimates. This suggests that metaheuristic approaches like BKA help avoid local optima and improve convergence speed when dealing with truncated likelihood functions. For small sample sizes where numerical instability and local optima are more problematic, BFGS-BKA provides meaningful improvements. For larger datasets, while all methods perform well, BFGS-BKA still maintains a measurable edge in accuracy.

Figures 1–3 show the performance of Newton's, DFP, BFGS, and BFGS-BKA algorithms in terms of time that represents the average computation time (in seconds) used by the all algorithms in the case of successful runs. We can see how the proposed algorithm is better than the others because of the Cauchy mutation that causes better exploration and the leader strategy that causes faster convergence leading to a balance between global and local search.

11 Conclusions

This paper addressed the critical challenge of accurately estimating parameters in truncated Poisson regression models, where standard maximum likelihood estimation is complicated by a truncated likelihood function that includes a non-trivial normalizing constant. The paper proposed the use of the BKO algorithm, a metaheuristic inspired by the hunting and migratory behavior of black kites, which aims to enhance exploration and exploitation capabilities when searching the parameter space. By leveraging BKO, the study seeks to improve the stability, convergence speed, and accuracy of parameter estimates in truncated Poisson models. Simulation studies and empirical analyses included in the paper demonstrate the superior performance of this approach compared to classical optimization methods, highlighting its potential as a robust and efficient solution for parameter estimation in truncated count data settings.

Data availability statement

The data analyzed in this study is subject to the following licenses/restrictions: the data will be under request. Requests to access these datasets should be directed to Zakariya Algamal, zakariya.algamal@uomosul.edu.iq.

Author contributions

GB: Conceptualization, Formal analysis, Software, Writing – original draft, Writing – review & editing. SW: Conceptualization, Formal analysis, Methodology, Resources, Writing – original draft, Writing – review & editing. ZA: Supervision, Validation, Writing – review & editing.

Funding

The author(s) declared that financial support was not received for this work and/or its publication.

Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

The author(s) declared that generative AI was not used in the creation of this manuscript.

Any alternative text (alt text) provided alongside figures in this article has been generated by Frontiers with the support of artificial intelligence and reasonable efforts have been made to ensure accuracy, including review by the authors wherever possible. If you identify any issues, please contact us.

Publisher's note

References

1. Ahmed Alangood HN, Algamal Z, Khaleel MA. *Variable Selection in Poisson Regression Model based on Chaotic Meta-Heuristic Search Algorithm*. BIO Web of Conferences, 97 (2024). doi: 10.1051/bioconf/20249700161

2. Algamal Z. *Variable Selection in Count Data Regression Model based on Firefly Algorithm*. Statistics, Optimization and Information Computing, 7 (2019). doi: 10.19139/soic.v7i2.566

3. Alwani ZIZ, Ibrahim AI, Yunus RM, Yusof F. Application of zero-truncated count data regression models to air-pollution disease. *J Phys Conf Ser.* (2021) 1988:012096. doi: 10.1088/1742-6596/1988/1/012096

4. Hilbe J-M. *Modeling-Count-Data*. Cambridge: Cambridge University Press (2014). doi: 10.1017/CBO9781139236065

5. Hawa NS, Mustafa MY, Kibria BMG, Algamal ZY. Bootstrap Liu-type estimator for Conway-Maxwell-Poisson regression model. *Commun Stat Simul Comput.* (2025) 1–12. doi: 10.1080/03610918.2025.2462680

6. Kara R, Yeşilova A. Zero truncated models in regression analysis: an examination of their advantages on small mean values. *J Inst Nat Appl Sci.* (2025) 30:102–12. doi: 10.53433/yyufbed.1590611

7. Alkhateeb A, Algamal Z. Jackknifed liu-type estimator in Poisson regression model. *J Iran Stat Soc.* (2020) 19:21–37. doi: 10.29252/jirss.19.1.21

8. Liu Y, Li W, Zhang X. A marginalized zero-truncated Poisson regression model and its model averaging prediction. *Commun Math Stat.* (2025) 13:527–70. doi: 10.1007/s40304-022-00312-8

9. Wani MK, Ahmad PB. One-inflated zero-truncated Poisson distribution: statistical properties and real life applications. *Ann Data Sci.* (2025) 12:639–66. doi: 10.1007/s40745-024-00526-3

10. Young DS, Roemmele ES, Yeh P. Zero-inflated modeling part I: traditional zero-inflated count regression models, their applications, and computational tools. *WIREs Comput Stat.* (2022) 14:e1541. doi: 10.1002/wics.1541

11. Qi Y, Yu-Zhu T, Yi-Jing Z, Yue W, Zhi-Bao M. An ordinal collaboration network model with zero truncated poisson latent variables and its application. *Stat* (2025) 14:e70040. doi: 10.1002/sta4.70040

12. Ünlü HK, Young DS, Yigiter A, Hilal Özcebe L. A mixture model with Poisson and zero-truncated Poisson components to analyze road traffic accidents in Turkey. *J Appl Stat.* (2022) 49:1003–17. doi: 10.1080/02664763.2020.1843610

13. Rao SS. *Engineering Optimization Theory and Practice.* Hoboken, NJ: John Wiley & Sons, Inc. (2009).

14. Yu X, Gen M. *Introduction to Evolutionary Algorithms.* London: Springer-Verlag London Limited (2010). doi: 10.1109/ICCIE.2010.5668407

15. Wang J, Wang WC, Hu XX, Qiu L, Zang HF. Black-winged kite algorithm: a nature-inspired meta-heuristic for solving benchmark functions and engineering problems. *Artif Intell Rev.* (2024) 57:98. doi: 10.1007/s10462-024-10723-4

16. Zhou H, Alexander D, Lange K. A quasi-Newton acceleration for high-dimensional optimization algorithms. *Stat Comput.* (2011) 21:261–73. doi: 10.1007/s11222-009-9166-3

17. Jameel MS, Basheer GT, Al-Bayati AY, Algamal ZY. Parameter estimation of a truncated regression model based on improving numerical optimization algorithms. *J Phys Conf Ser.* (2021) 1897:012059. doi: 10.1088/1742-6596/1897/1/012059

18. Hwang WH, Stoklosa J, Wang CY. Population size estimation using zero-truncated Poisson regression with measurement error. *J Agric Biol Environ Stat.* (2022) 27:303–20. doi: 10.1007/s13253-021-00481-z

19. Green W. *Econometric Analysis.* 5th ed. Upper Saddle River, NJ: Prentice Hall (2003).

20. Demaris A. *Regression with Social Data Modeling Continuous and Limited Response Variables.* New York, NY: John Wiley & Sons, Inc. (2004). doi: 10.1002/0471677566

21. Heij C, de Boer P, Franses PH, Kloek T, van Dijk HK. *Econometric Methods with Applications in Business and Economics*. Oxford: Oxford University Press (2004).

22. Umar MA, Jimoh K, Yahya WB. *A Note on the Applications of some Zero Truncated.* Professional Statisticians Society of Nigeria, 3 (2019).

23. Li X, Sun Y, Tian G, Liang J, Shi J. Mean regression model for the zero-truncated Poisson distribution and its generalization. *Comput Stat Data Anal.* (2023) 179:107650. doi: 10.1016/j.csda.2022.107650

24. Rahman S. *Truncated Distributions and their Applications, 2004-2005* (2004).

25. Mwandigha LM, Fraser KJ, Racine-Poon A, Mouksassi MS, Ghani AC. Power calculations for cluster randomized trials (CRTs) with right-truncated Poisson-distributed outcomes: a motivating example from a malaria vector control trial. *Int J Epidemiol.* (2020) 49:954–62. doi: 10.1093/ije/dyz277

26. Suaiee AM. *A Double Truncated Poisson Regression Model with Random Effects*. Greeley, CO: University of Northern Colorado (2013).

27. Karlsson M. Estimators of regression parameters for truncated and censored data. *Metrika* (2006) 63:329–41. doi: 10.1007/s00184-005-0023-x

28. Newey WK. Conditional moment restrictions in censored and truncated regression models. *Econ Theory* (2001) 17:863–88. doi: 10.1017/S0266466601175018

29. Irshad M, Chesneau C, Shibu DS, Monisha M, Maya R. Lagrangian zero truncated Poisson distribution: properties regression model and applications. *Symmetry* (2022) 14:1775. doi: 10.3390/sym14091775

30. Sabri Al-zubaidi M, Abdulah EK. Comparison of estimation methods for zero truncated poisson regression model. *J Econ Admin Sci.* (2024) 30:492–508. doi: 10.33095/7bt1f714

31. Kumar S, Singh BP, Madhusudan JV. Factors influencing migration: application of truncated poisson regression. *Int J Adv Sci Technol.* (2020) 29:9112–20.

32. Hussain EA, Al-Shallawi ANS, Saied HA. Using maximum likelihood method to estimate parameters of the linear regression T truncated model. *NTU J Pure Sci.* (2022) 4:26–34. doi: 10.56286/ntujps.v1i4.313

33. Luksan L, Spedicatob E. Variable metric methods for unconstrained optimization. *J Comput Appl Math.* (2000) 124:61–95. doi: 10.1016/S0377-0427(00)00420-9

34. Zhang Z, Wang X, Yue Y. Heuristic optimization algorithm of black-winged kite fused with osprey and its engineering application. *Biomimetics* (2024) 9:595. doi: 10.3390/biomimetics9100595

35. Zhao M, Su Z, Hua Z, Zhao C. Improved black-winged kite algorithm based on chaotic mapping and adversarial learning. *J Phys Conf Ser.* (2024) 2898:012040. doi: 10.1088/1742-6596/2898/1/012040

36. David FN, Johnson NL. The truncated poisson. *Biometrics.* (1952) 8:275–85.